**University of Leicester**

**Future 50 PhD Scholarship**

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| **Project Reference** | CMS Fasondini |

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**Section 2 – *Project Information***

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| **Project Title** | Sparse spectral methods on new geometries for differential equations | |
| **Project Highlights:** | 1. | Sparse spectral methods with new basis functions on new geometries |
| 2. | Implementation in open source, high-performance code |
| 3. | Numerous computational modelling applications |
| **Project Summary** | | |
| Spectral methods are used to compute solutions to differential and integral equations. Typically, this is done by expansions in orthogonal basis functions on the entire domain of the problem, or on multiple subdomains in a spectral element (or high-p-finite element) method.  Traditional spectral methods are well known to have excellent convergence properties for analytic solutions (exponentially fast convergence). However, this comes at the expense of solving dense and ill-conditioned linear systems. In recent years, spectral methods have been devised that result in sparse and well-conditioned linear systems that can be solved with fast, optimal complexity algorithms. Furthermore, these sparse spectral methods have been extended from intervals to certain regions in 2D and 3D, for example triangles, balls, cones, disks, disk slices, trapeziums and spherical caps (see, for example, [1]). In fact, sparse spectral methods can be designed for any geometry described by an algebraic curve or surface (zero sets of multivariate polynomials), however in practice one requires an explicit family of orthogonal basis functions or a stable and efficient computational method for constructing the basis functions.  The aim of this project is to design sparse spectral methods for a set of geometries described by algebraic curves and surfaces. The project supervisor and his collaborators have recently constructed new orthogonal polynomial basis functions (OPs) on a class of algebraic curves in 1D with a stable algorithm that has linear complexity [2,3]. Their methodology can be extended to construct OPs on regions in 2D bounded by the same class of algebraic curves and their associated surfaces of revolution in 3D. These OPs will be used in this project to construct sparse matrix representations of conversion (change-of-basis), multiplication and differentiation operators as well as transforms based on quadrature. These sparse matrices and transforms will be used to devise sparse spectral methods on the above-mentioned regions in 2D and 3D, which will be implemented in the open source Julia programming language. The resulting spectral methods will be tested on model problems that arise in acoustics and fluid mechanics (e.g., Laplace and Helmholtz problems). Ultimately, the goal is to extend these spectral methods to sparse spectral element methods for computationally challenging applications in numerical weather prediction, acoustic and elastic wave propagation and medical imaging.  [1] B. Snowball and S. Olver. Sparse spectral and p-finite element methods for partial differential equations on disk slices and trapeziums. *Stud. Appl. Math.*, 145:3–35, 2020.  [2] M. Fasondini, S. Olver, and Y. Xu. Orthogonal polynomials on planar cubic curves. *Found. Comput. Math.*, 1-31, 2021.  [3] M. Fasondini, S. Olver, and Y. Xu. Orthogonal polynomials on a class of planar algebraic curves. Submitted, <https://arxiv.org/abs/2211.06999> | | |