

# *Introduction to image formation and contrast transfer function (CTF)*

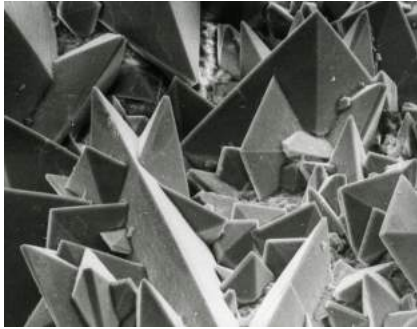
Marta Carroni, Swedish Cryo-EM Facility, Stockholm  
Midlands, Cryo-EM workshop, 16<sup>th</sup> March 2021



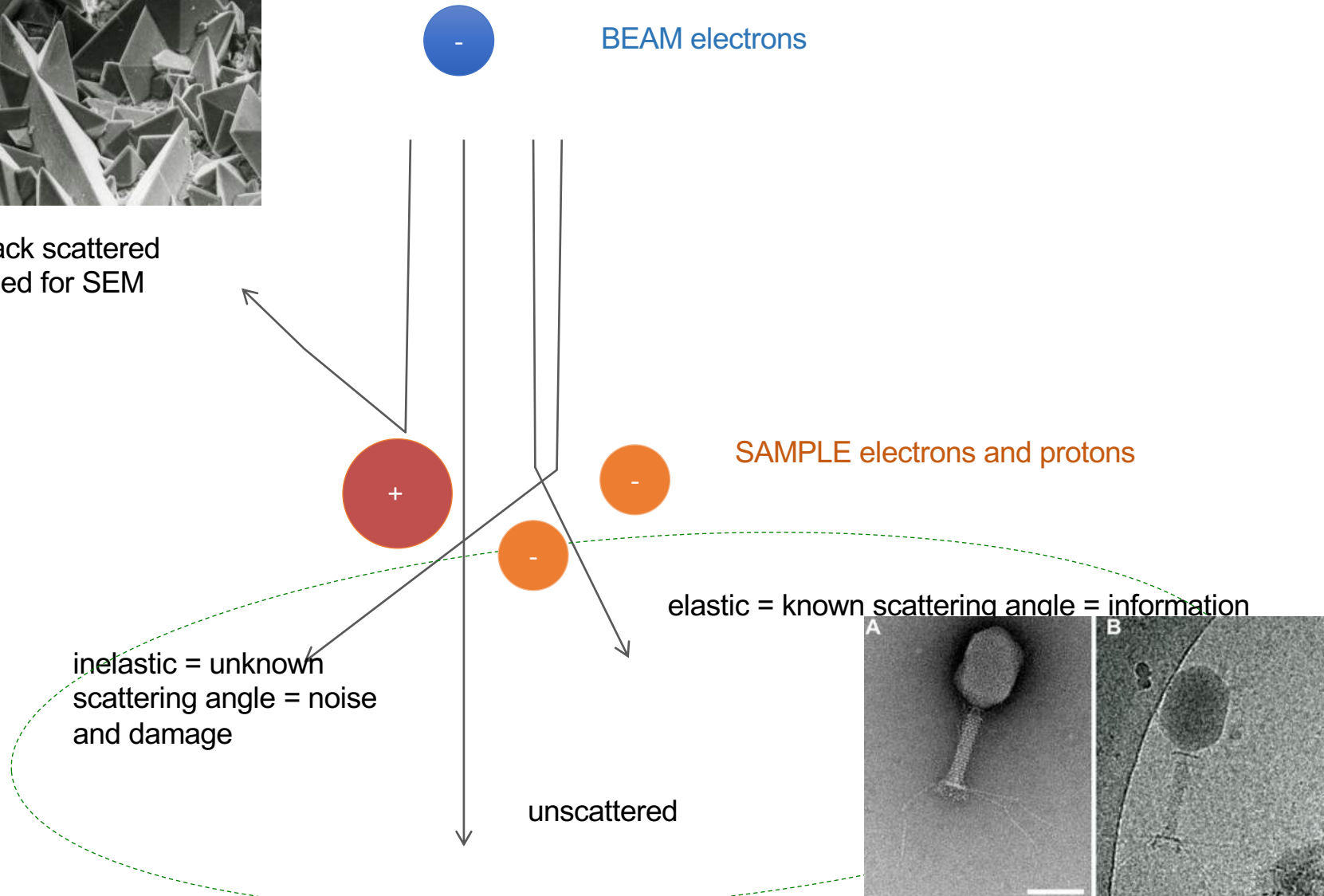
Stockholm  
University



# Electron-sample interaction



Back scattered  
used for SEM



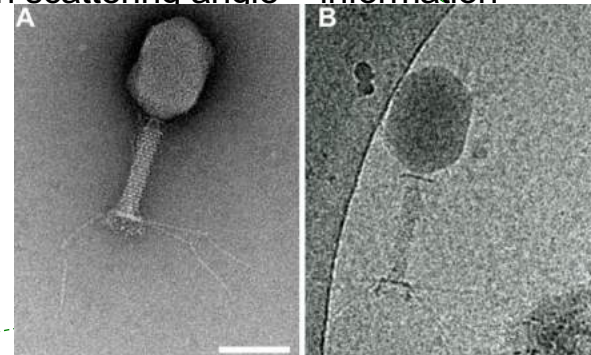
BEAM electrons

SAMPLE electrons and protons

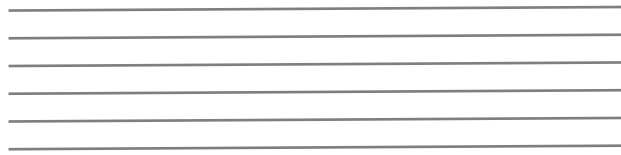
elastic = known scattering angle = information

inelastic = unknown  
scattering angle = noise  
and damage

unscattered



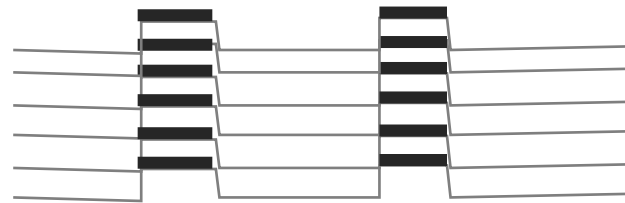
# Image formation: amplitude and phase contrast



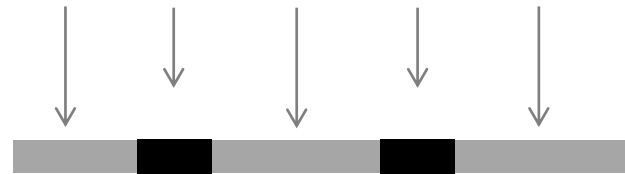
Incoming electron wave front



Sample

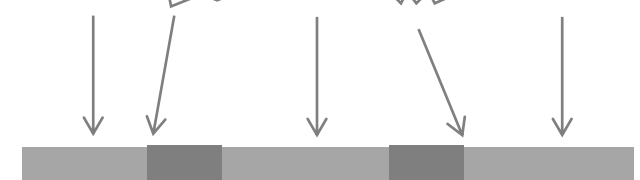
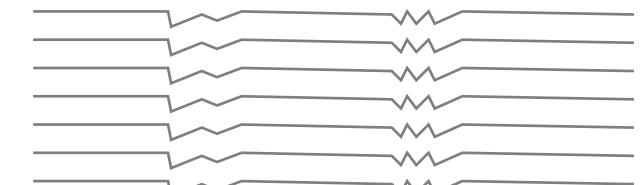
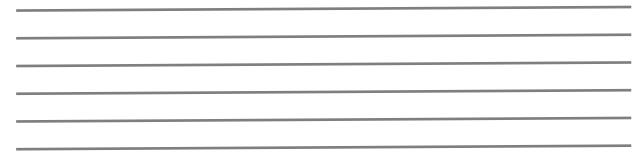


Outcoming electron wave front



## Amplitude contrast

resulting from the absorption of the incident beam

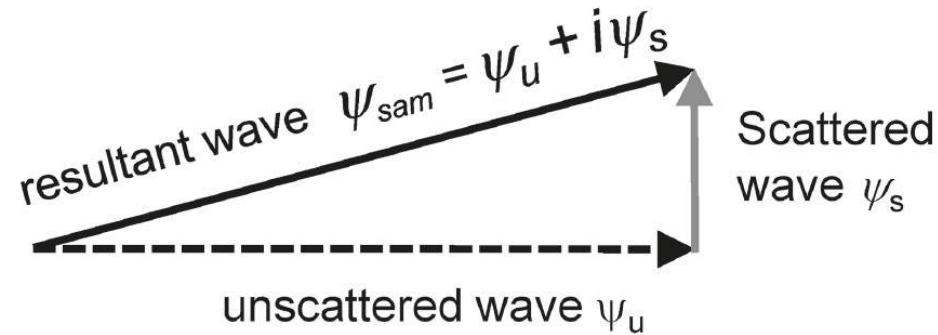


## Phase contrast

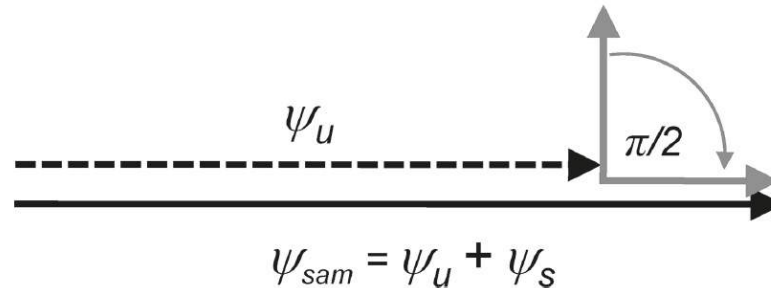
resulting from phase-shift of the incident beam

The amplitude contrast in cryo-EM biological sample is minimal (~10%). The contrast is achieved by “transforming” phase shift into amplitude contrast

# Increasing the contrast

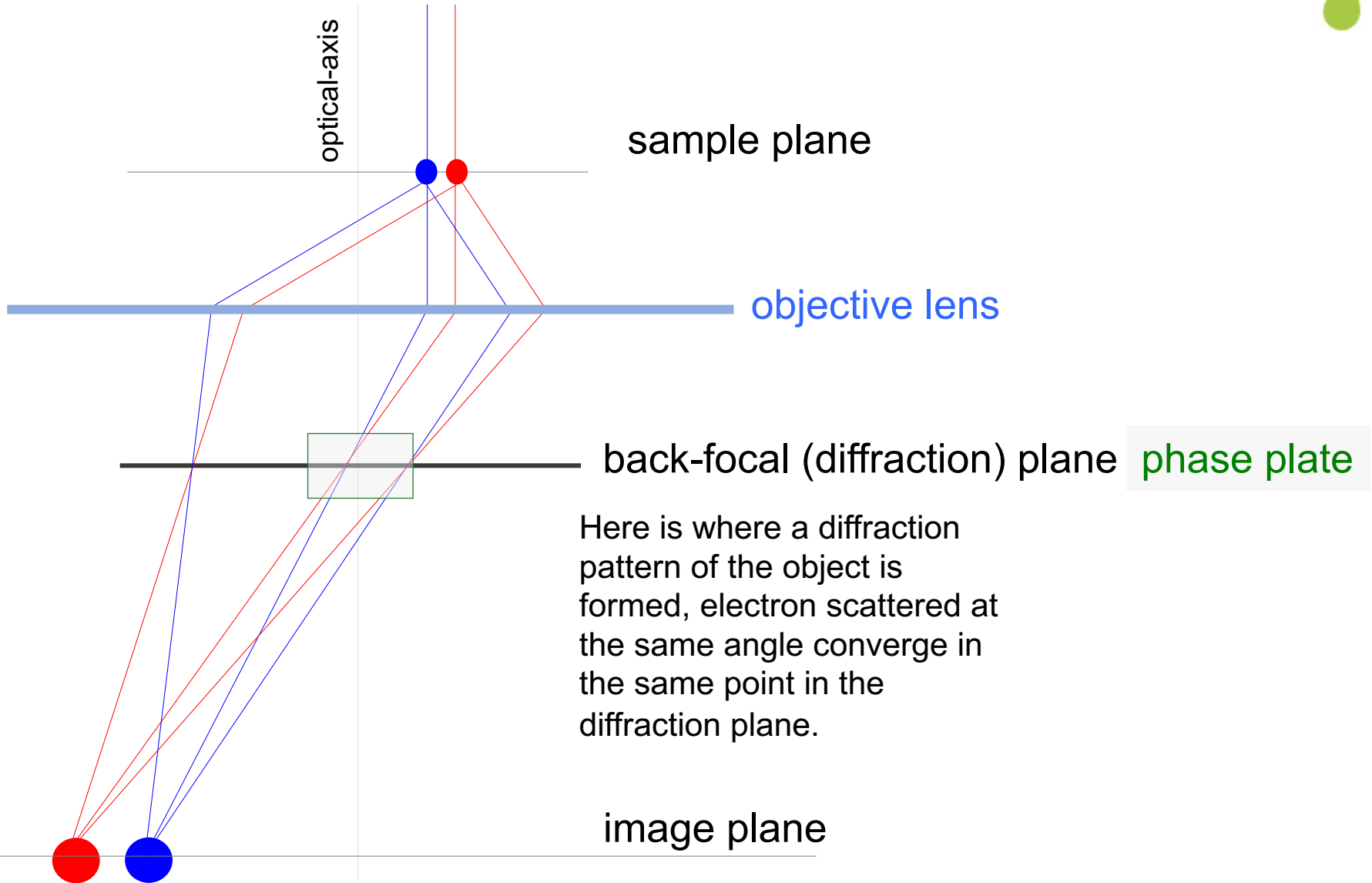


- The image is formed by the interference between the unscattered beam and the scattered one. The modules of the two vectors are not very different.

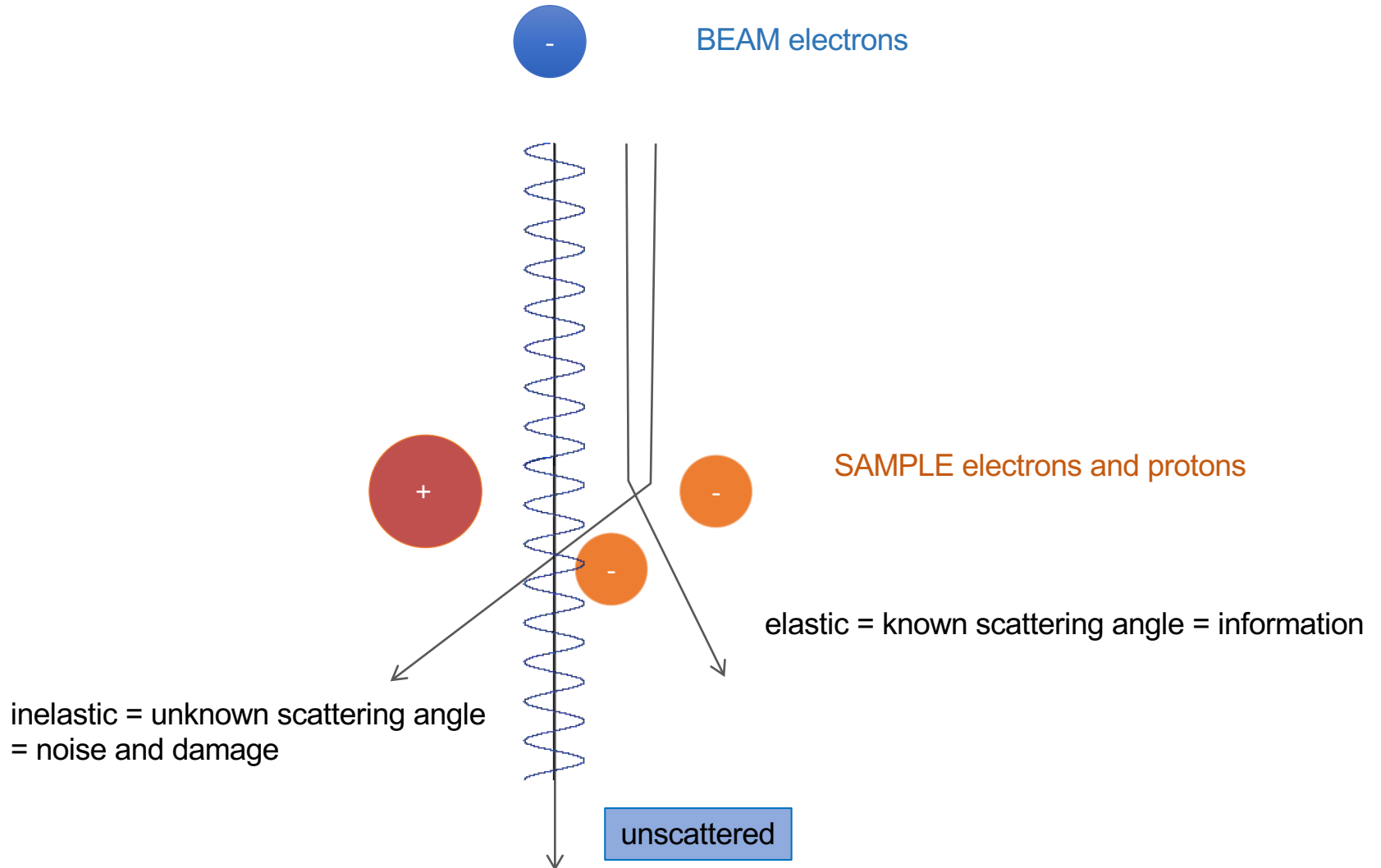


- We want to shift the phase of the scattered wave by  $90^\circ$  to rise the module of the resultant wave. This is for example achieved by using a phase plate.

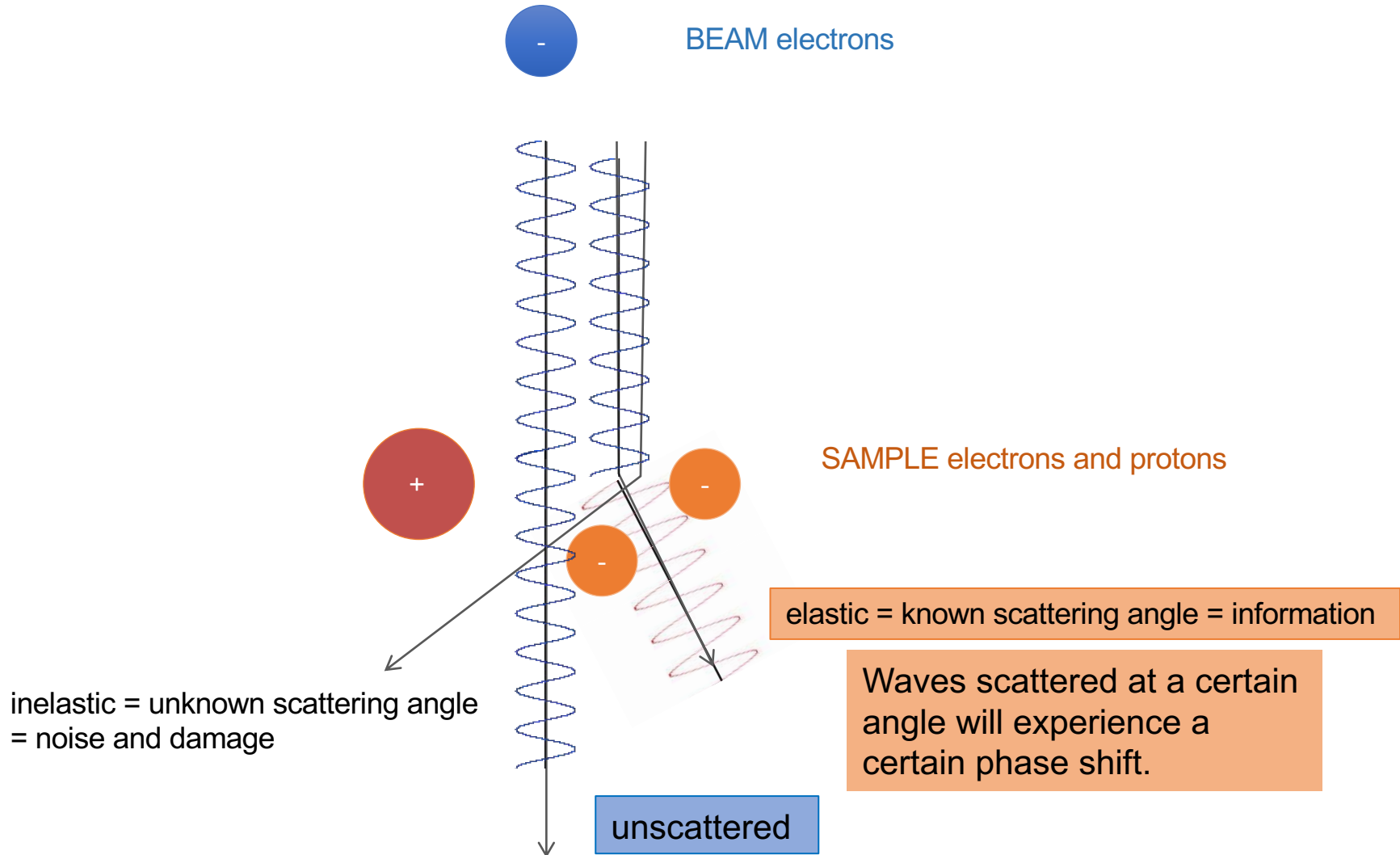
# Image formation



# Electron-sample interaction, wave paths



# Electron-sample interaction, wave paths



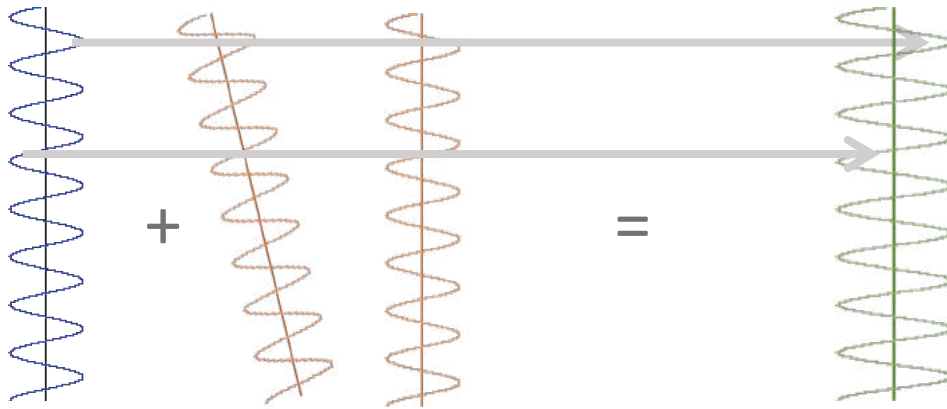
# Electron-sample interaction, wave paths

The final image is a combination of the unscattered and the scattered beam.

For each angle of scattering we have a certain phase shift.

When this phase shifted wave is combined with the unscattered wave a certain type of interference occurs.

When the interference is constructive the contrast will increase when destructive will decrease or will be equal to zero.



unscattered

Scattered at  
angle theta

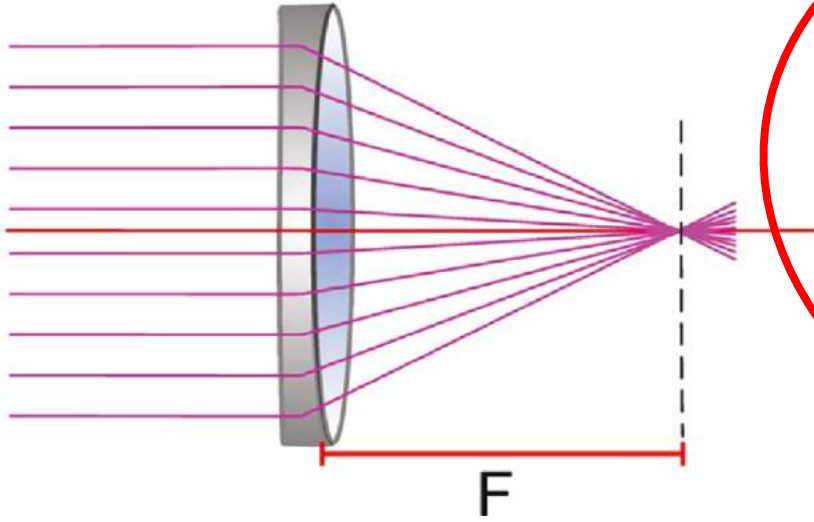
Constructive interference > more  
amplitude > more contrast



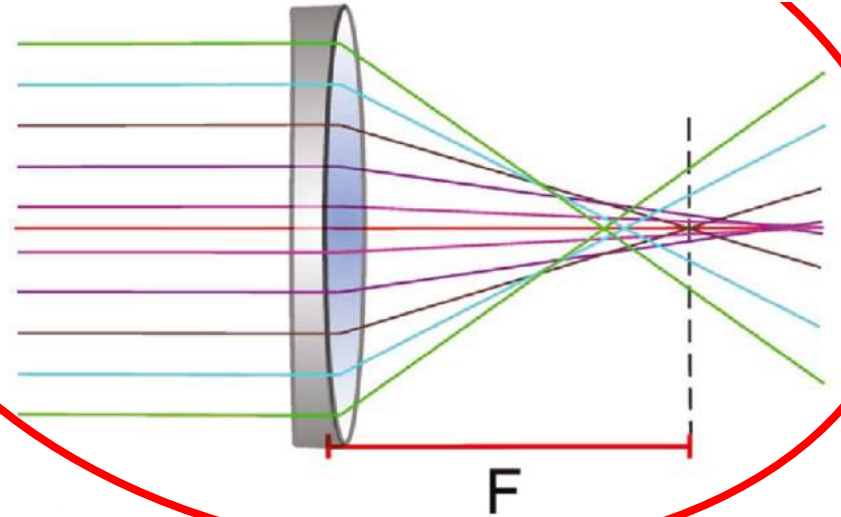
# Lens aberrations



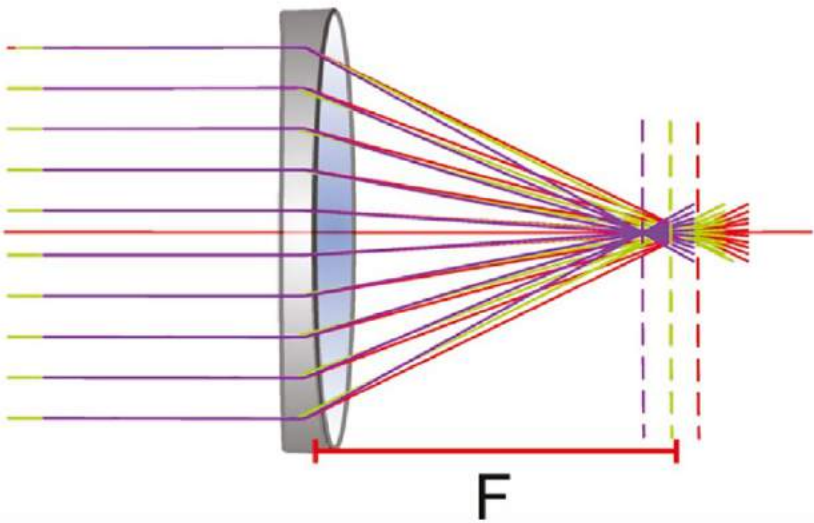
perfect lens



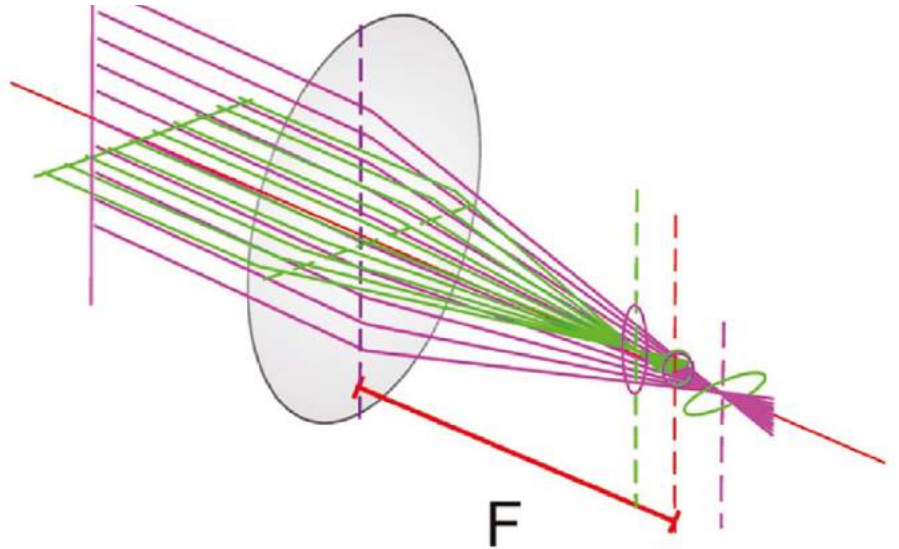
spherical aberration (Cs)



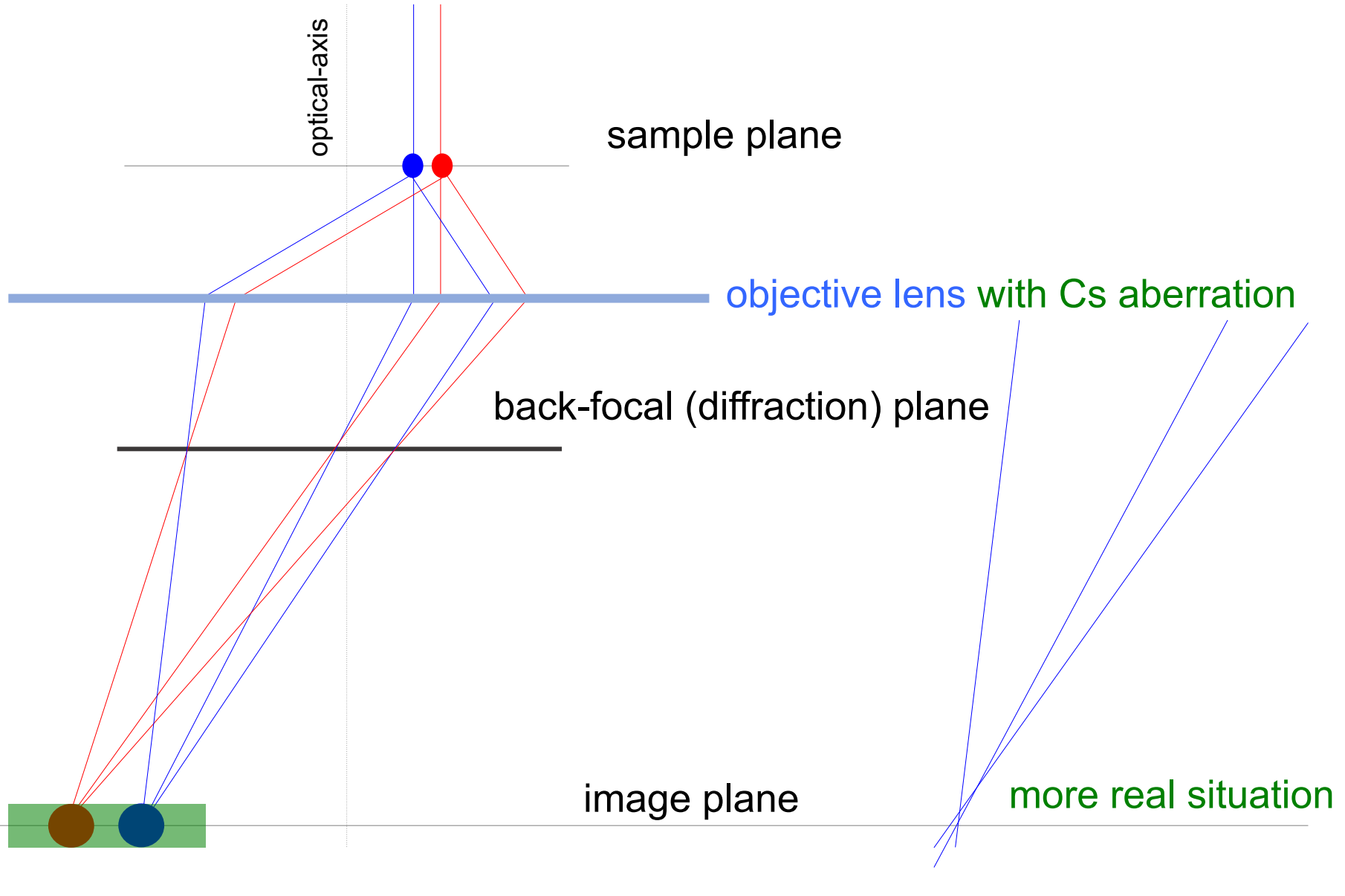
chromatic aberration (Cc)



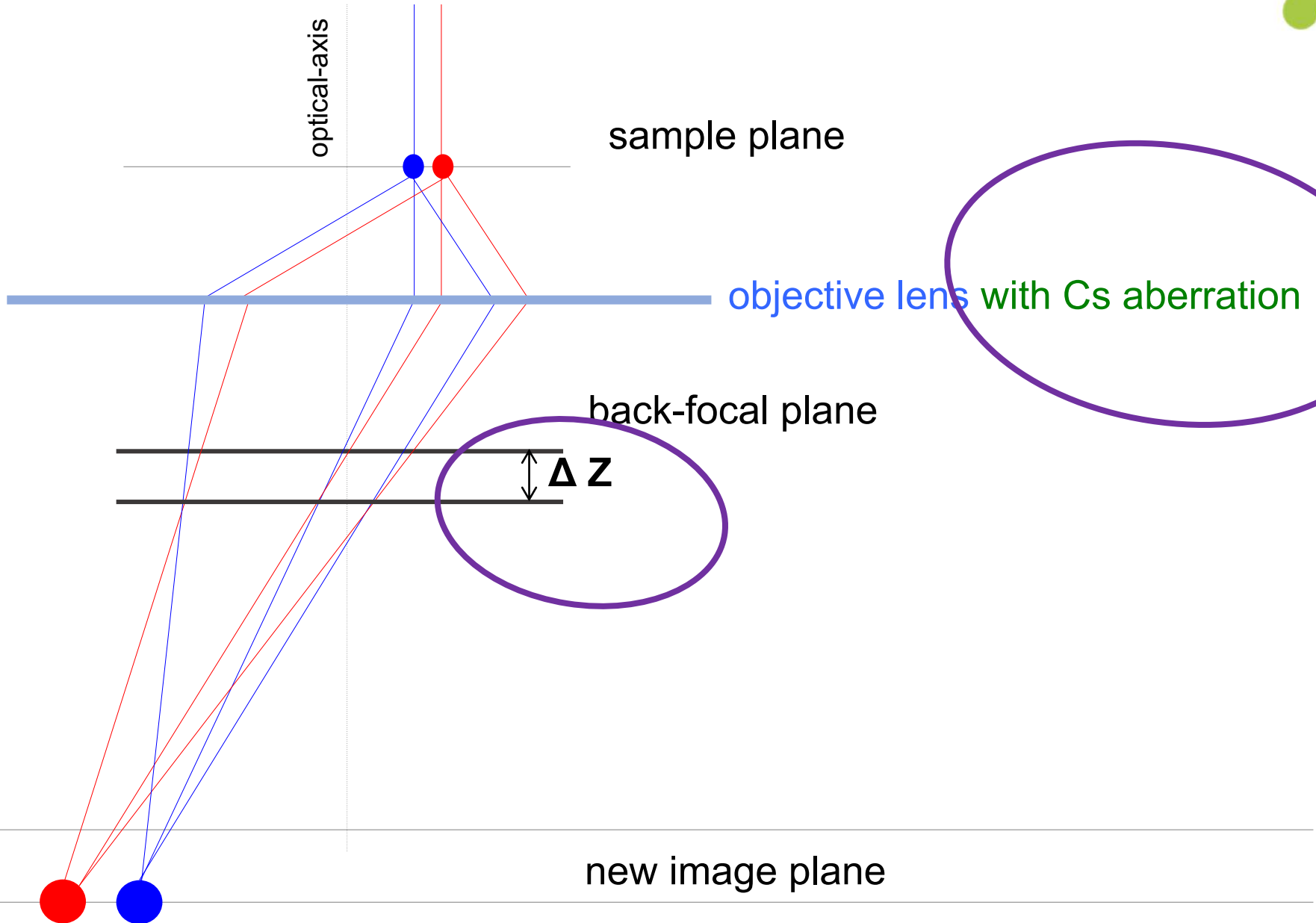
astigmatism



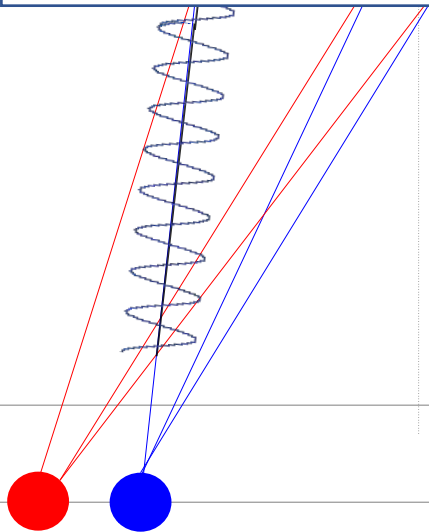
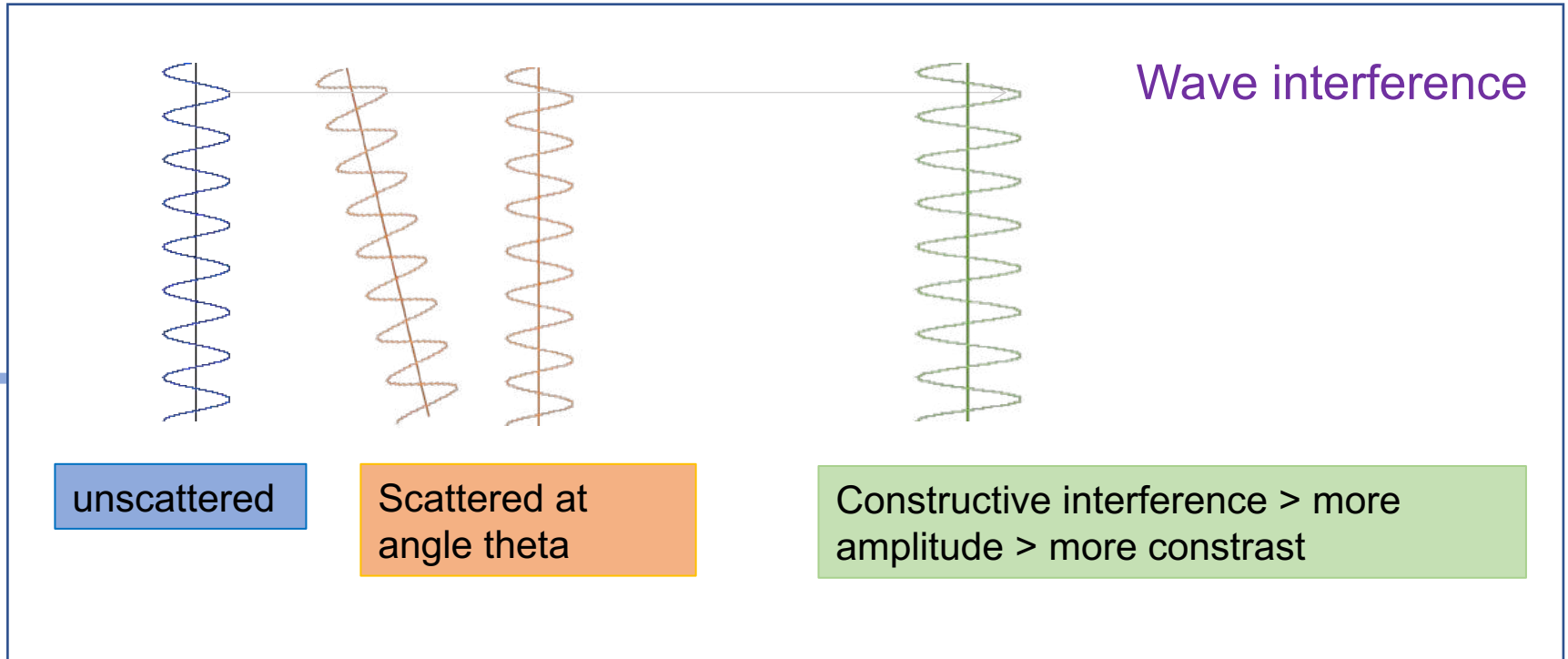
# Image formation, putting together all aberrations



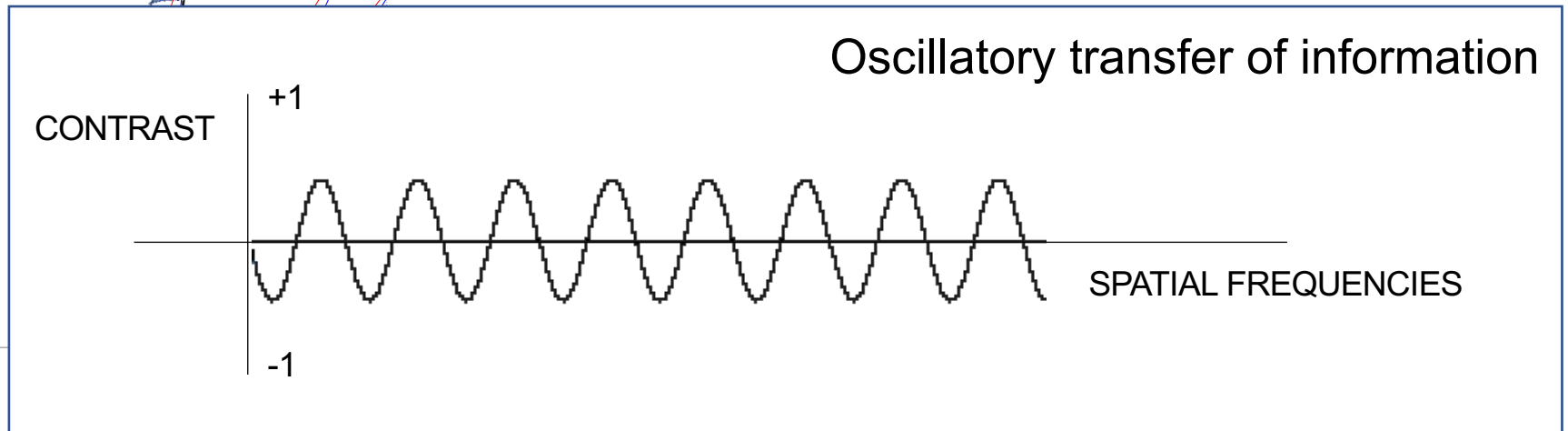
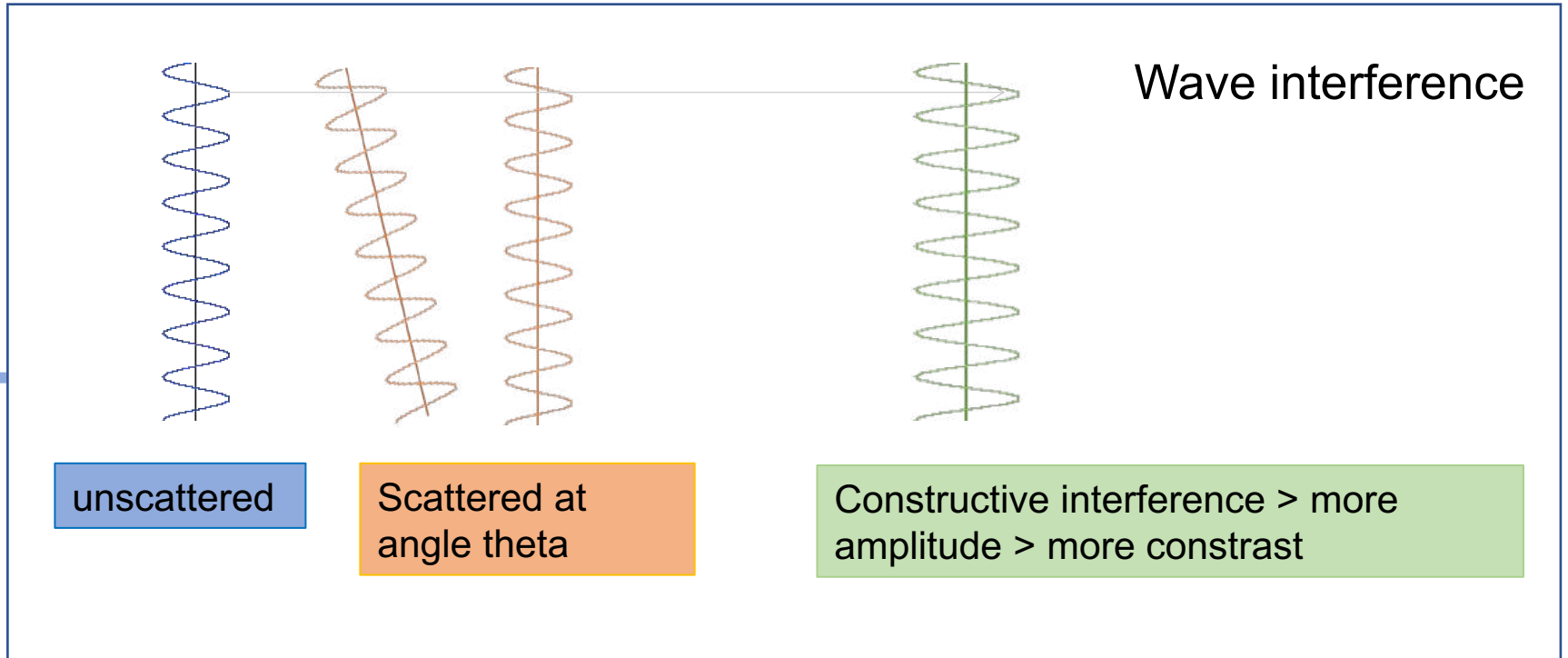
# Image formation, ray paths, Cs



# Image formation, putting together all aberrations

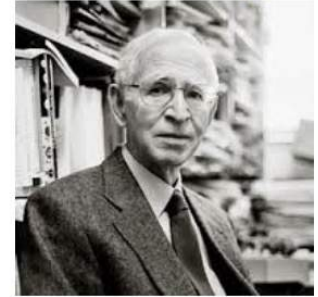


# Image formation (waves paths)



# Signal modulation by aberrations

Aaron Klug



The modulation of amplitudes of the scattered wave by lens aberrations is easily described at the diffraction plane as a function of the scattering angle (i.e. of the spatial frequency that is represented).

$$CTF(f) = A \sin(\underbrace{\pi\lambda\Delta z}_{\text{defocus}} f^2 - \frac{1}{2} \pi\lambda^3 \underbrace{c_s}_{\text{spherical aberration}} f^4)$$

scattering angle

wavelength

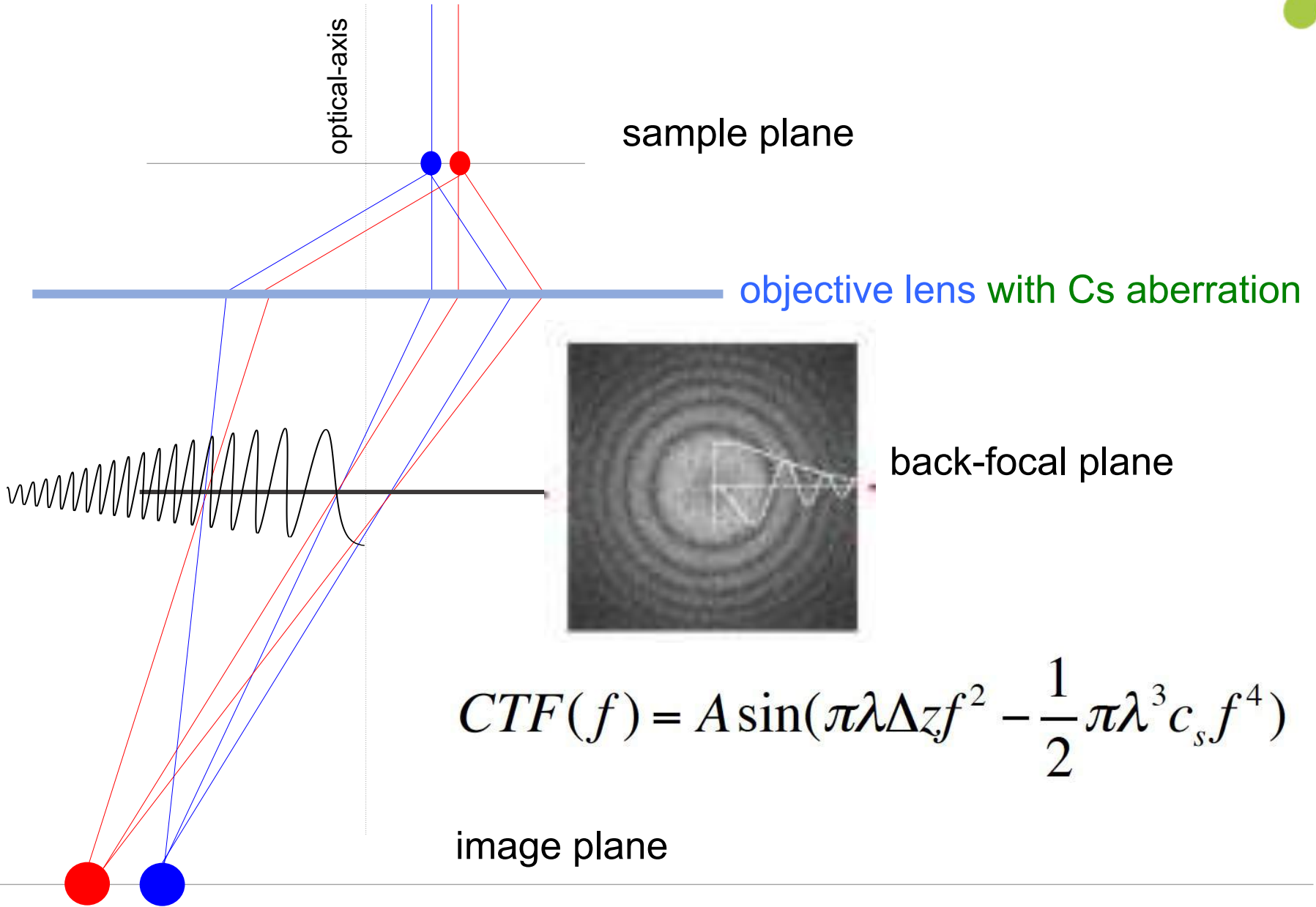
*Phil. Trans. Roy. Soc. Lond. B.* **261**, 105–118 (1971) [ 105 ]

*Printed in Great Britain*

Measurement and compensation of defocusing and aberrations by  
Fourier processing of electron micrographs

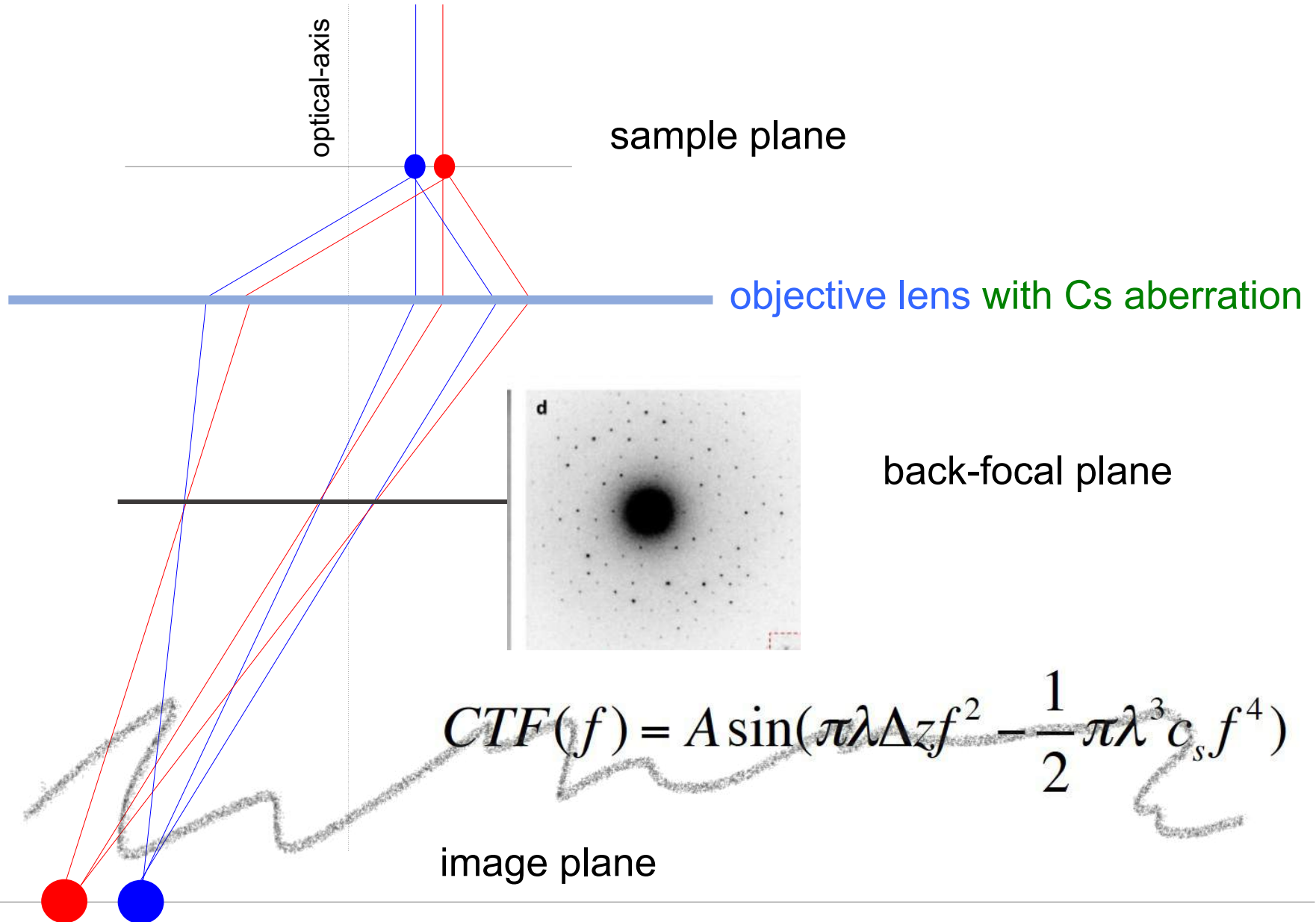
BY H. P. ERICKSON AND A. KLUG, F.R.S.  
*Medical Research Council Laboratory of Molecular Biology, Cambridge*

# The image formed is affected by the CTF



$$CTF(f) = A \sin(\pi \lambda \Delta_z f^2 - \frac{1}{2} \pi \lambda^3 c_s f^4)$$

# The diffraction pattern is not affected by the CTF



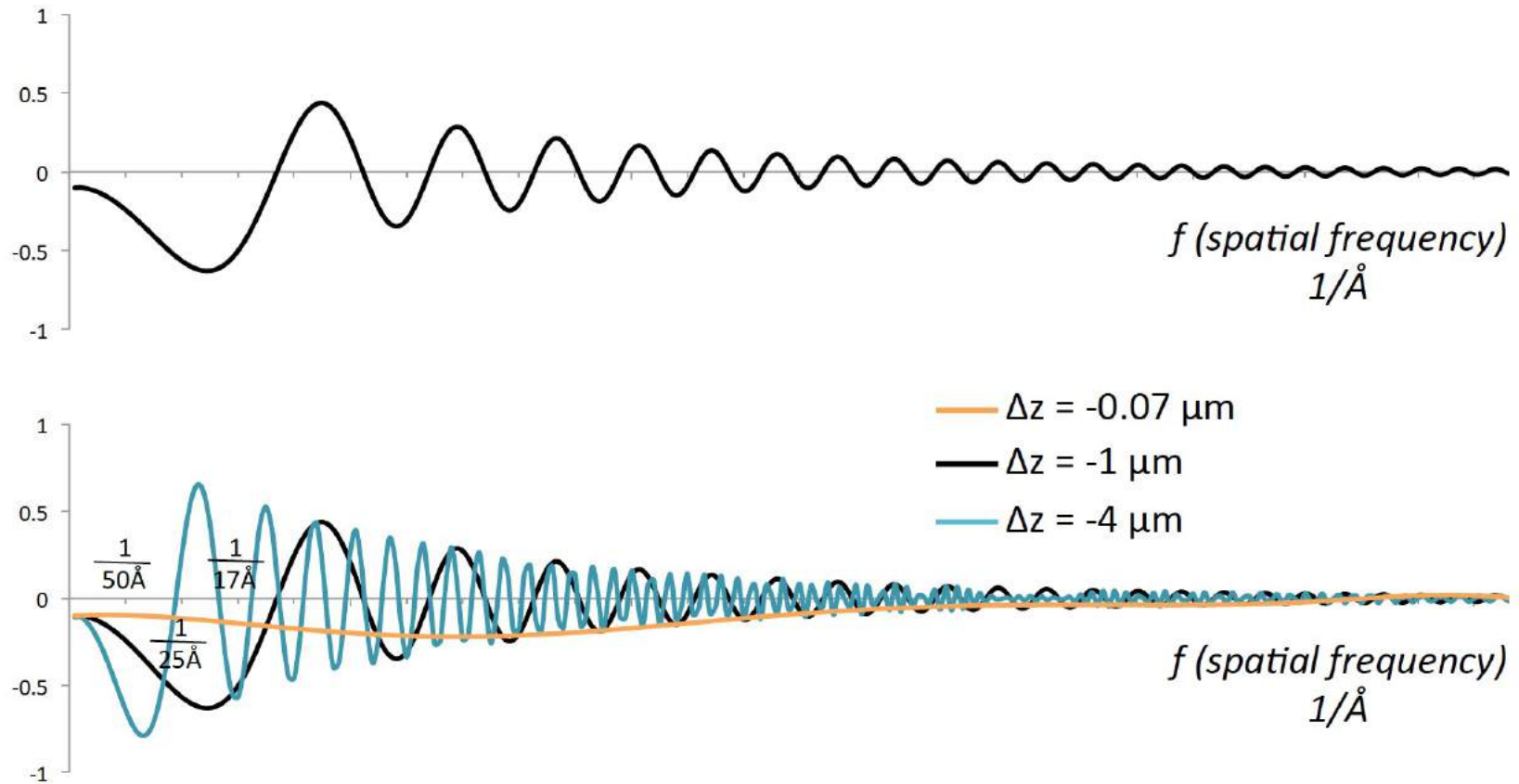


# At different defoci different zeros and inversions

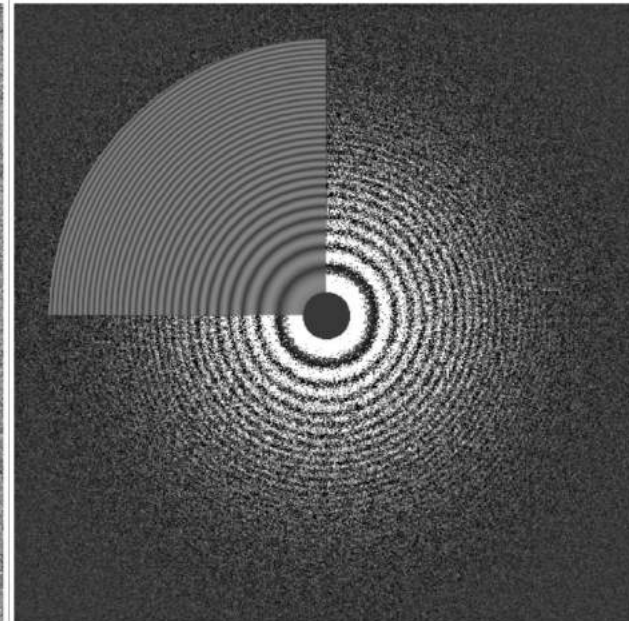
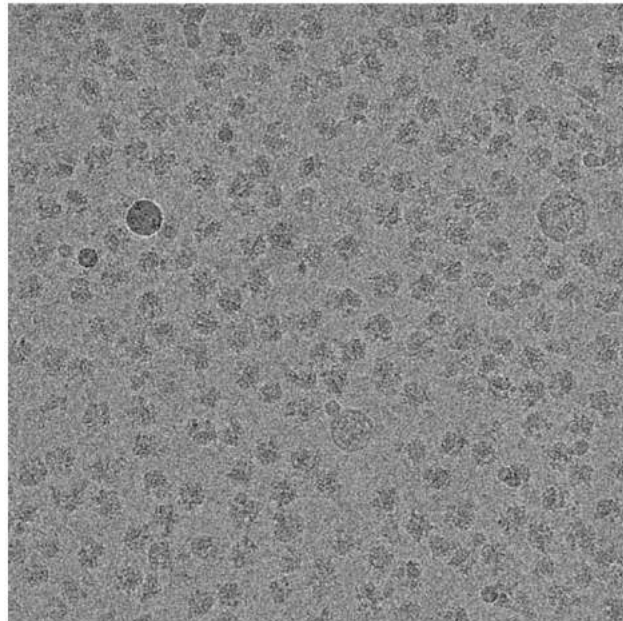
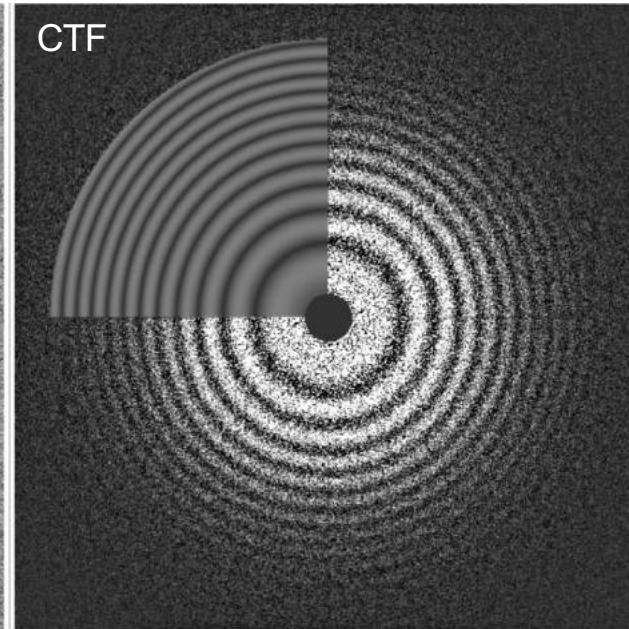
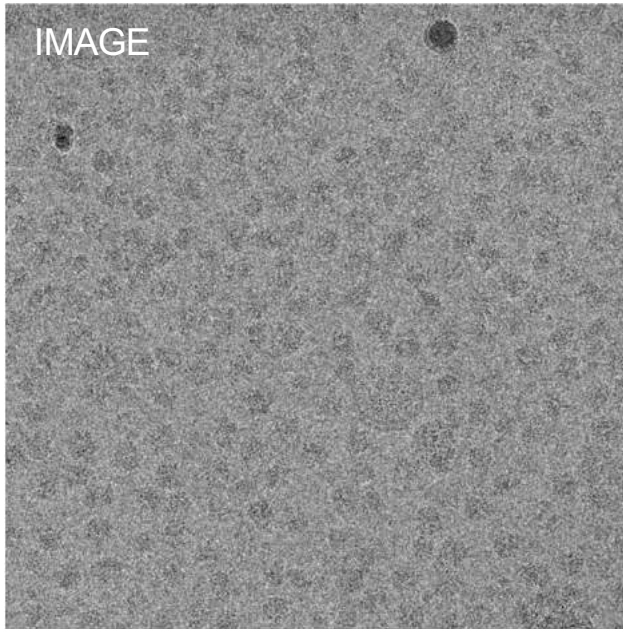
$$CTF(f) = A \sin(\pi \lambda \Delta z f^2 - \frac{1}{2} \pi \lambda^3 c_s f^4)$$

defocus

The oscillation of the CTF depends on the defocus

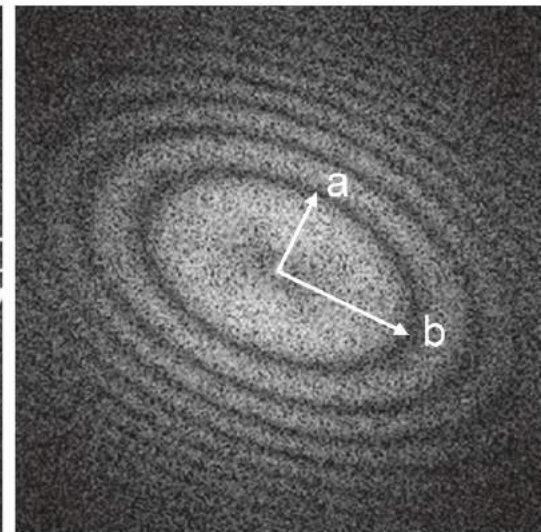
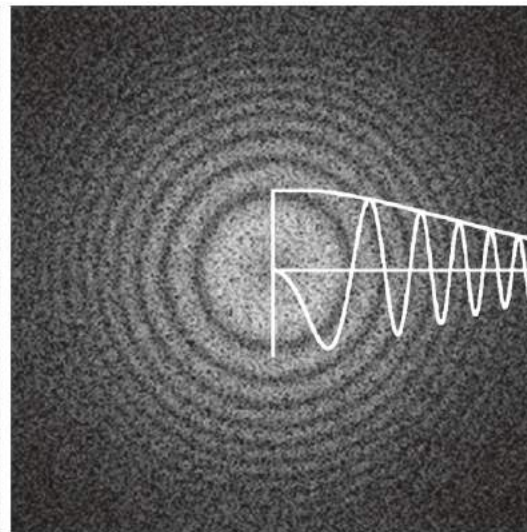
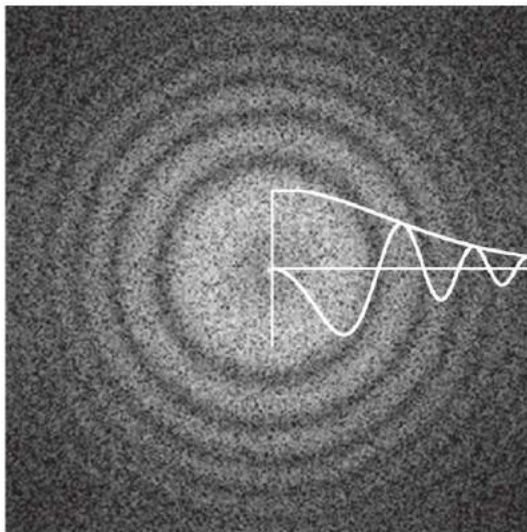
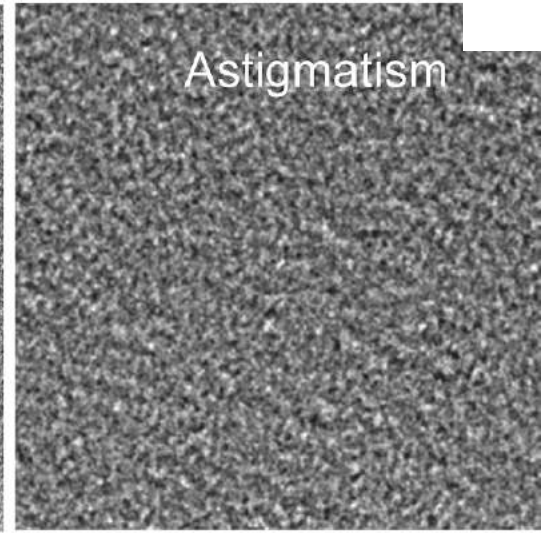
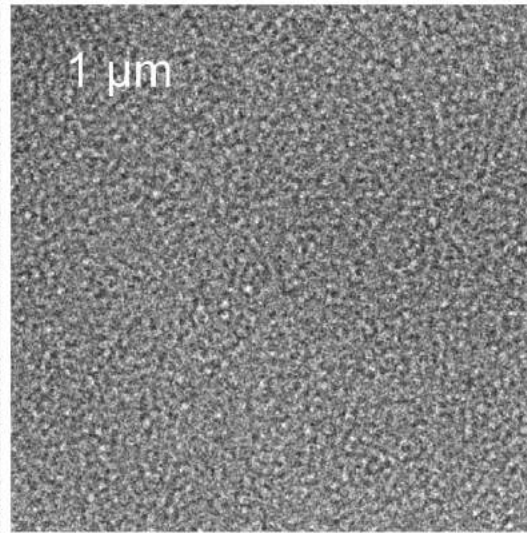
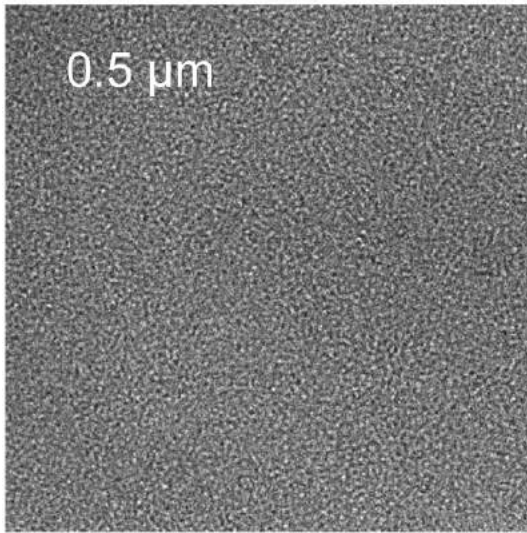


# Defocus effects



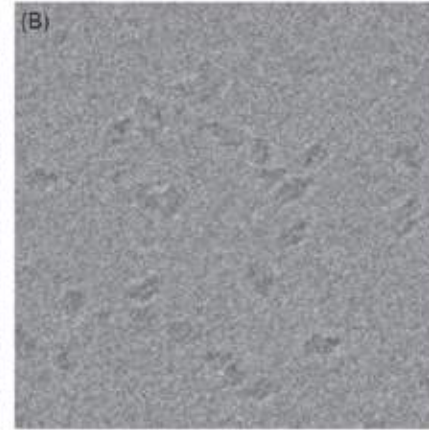
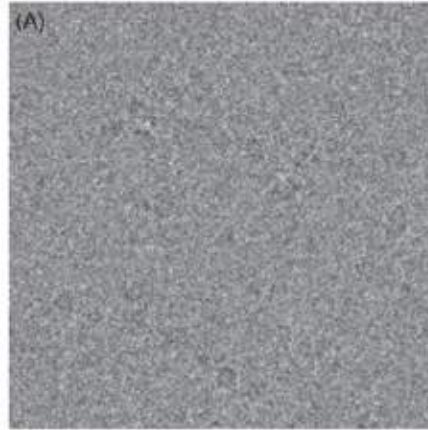


# Information from the power spectrum



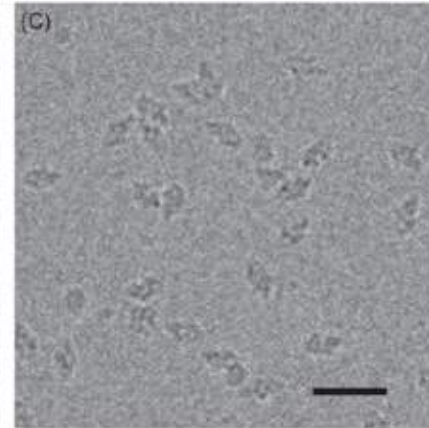
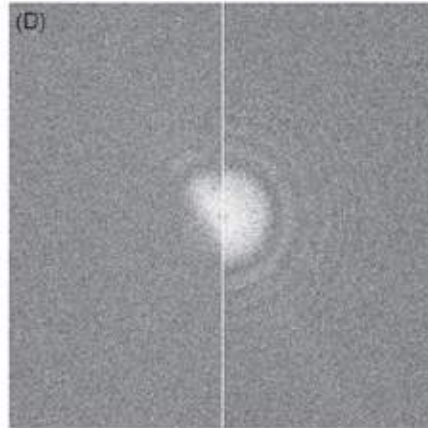
# Information from the power spectrum, drift

single frame with an electron dose of 6 electrons/Å<sup>2</sup>, the sum of movie frames with a total electron dose of 50 electrons/Å<sup>2</sup>



Without alignment

the power spectra (Fourier transforms) of the unaligned (left) and aligned (right) images



With alignment

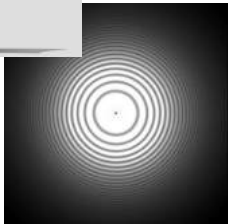
From Richard Henderson



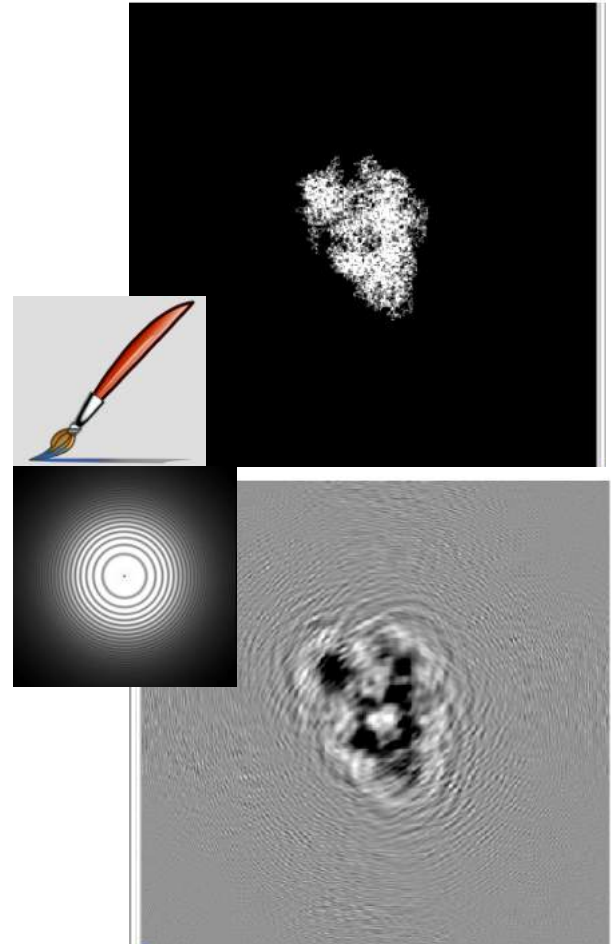
# CTF, the microscope brush



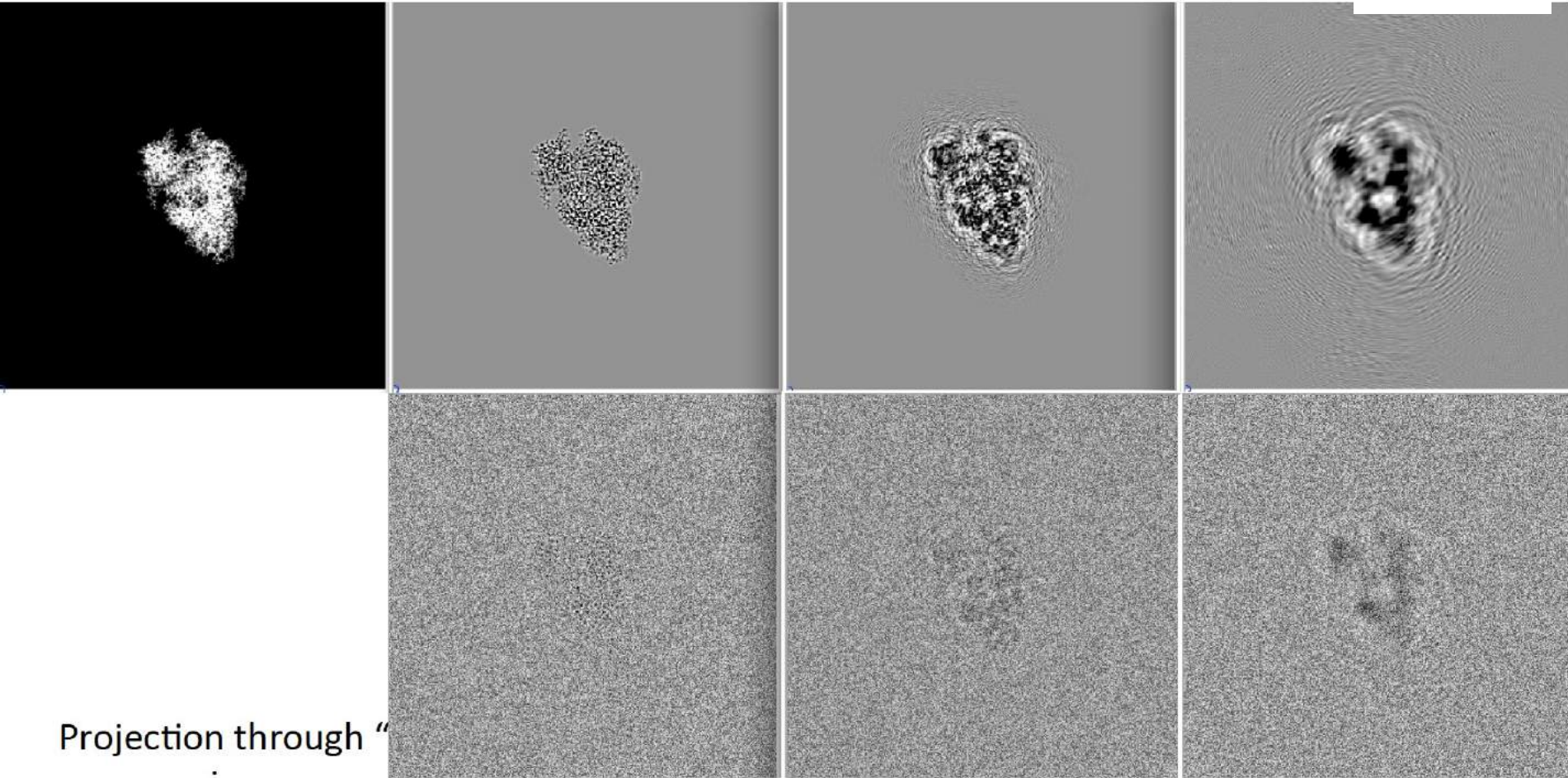
 correction



Microscope aberrations and ways of improving contrast affects the resulting image by the contrast transfer function (CTF).  
The CTF is the EM image point-spread function.

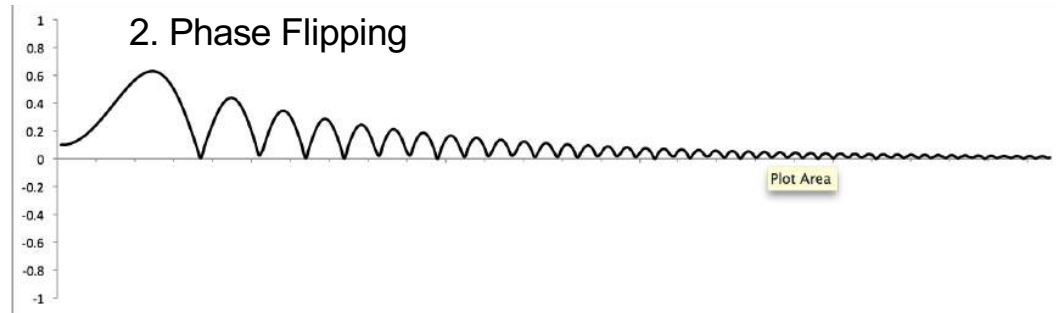
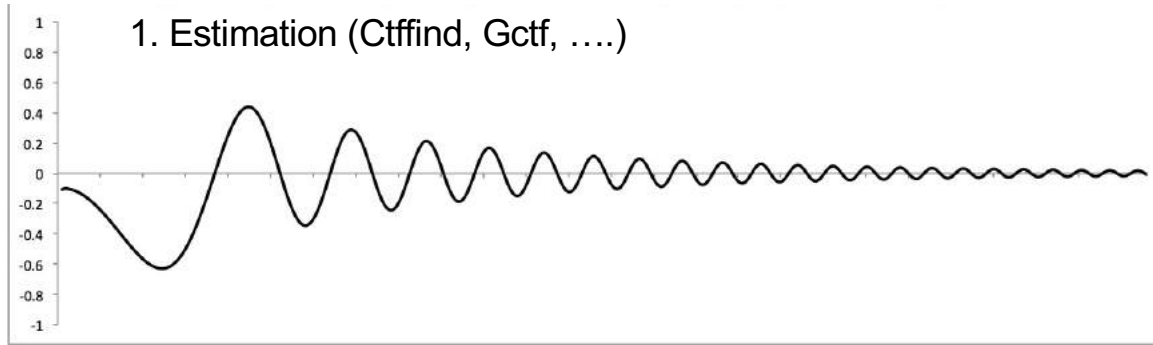
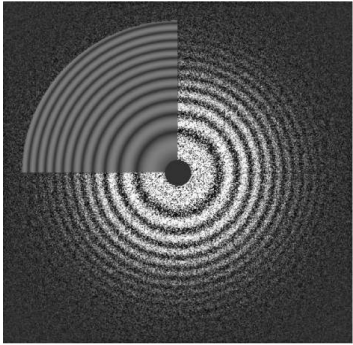


# CTF, useful at low signal to noise

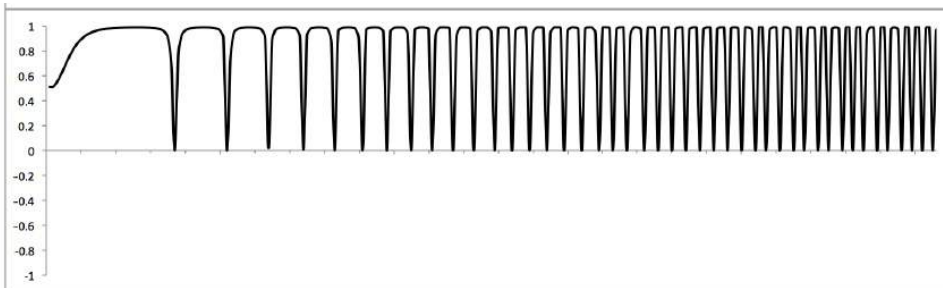


Projection through “

# CTF correction



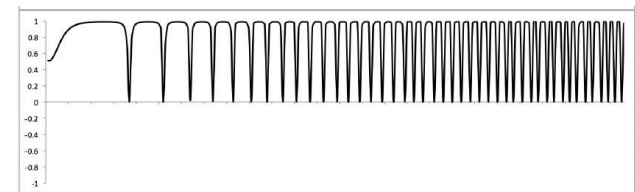
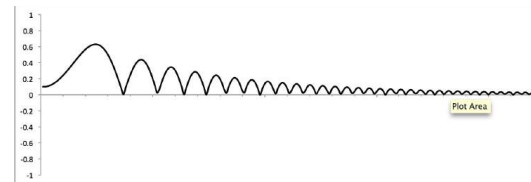
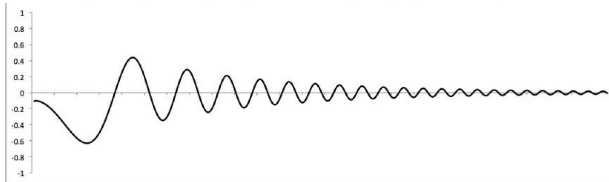
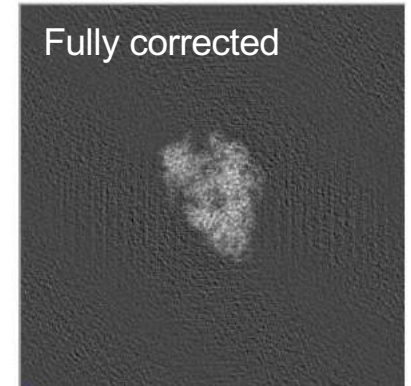
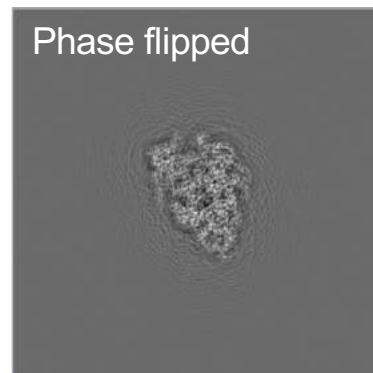
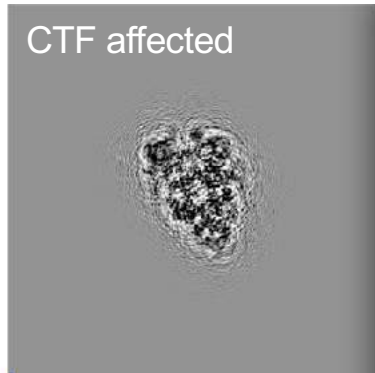
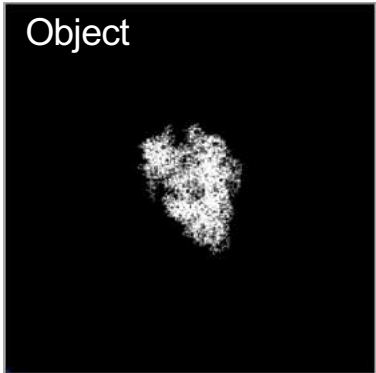
### 3. Full correction with Wiener filtering



$$FT_{corr} = FT \frac{CTF(f)}{CTF(f)^2 + \sigma_n}$$

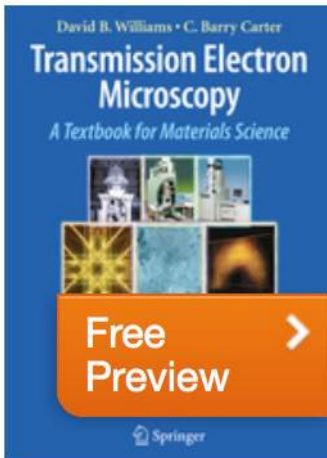
noise variance

# CTF correction





# Some literature



© 2009

## Transmission Electron Microscopy

A Textbook for Materials Science

Authors: **Williams**, David B., **Carter**, C. Barry

Henderson. The potential and limitations of neutrons, electrons and X-rays for atomic resolution microscopy of unstained biological samples (1995) *Quarterly Reviews in Biophysics* 28(2): 171-193.

Grant and Grigorieff. Measuring the optimal exposure for single particle cryo-EM using a 2.6 Å reconstruction of rotavirus (2015) *Elife* 4:e06980.

Baker and Rubinstein. Cryo-EM Part A: Chapter 15 -Radiation damage in electron cryomicroscopy (2010) *Methods in Enzymology*, 481:371-388.