The Department of Mathematics, Schools Newsletter, No.5

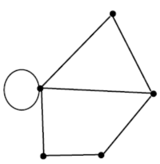
The unreasonable effectiveness of mathematics!

Mathematicians of course will naturally claim that their subject is the queen of the sciences. But scientists also have had this perception from as long ago as the mid twentieth century. In 1960 Eugene Wigner (top photograph), who was later awarded the physics Nobel prize, wrote an article entitled *The unreasonable effectiveness of mathematics in the natural sciences* in which he stated: "The enormous usefulness of mathematics in the natural sciences is something bordering on the mysterious and that there is no rational explanation for it ......The miracle of the appropriateness of the language of mathematics for the formulation of the laws of physics is a wonderful gift."

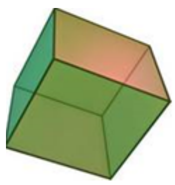
Some years after Wigner's article first appeared the scientist Richard Hamming (lower photograph), who was awarded the prestigious Turing Prize in 1968, wrote in his 1980 article *The unreasonable effectiveness of mathematics*: "From all of this I am forced to conclude both that mathematics is unreasonably effective ..... mathematics, mainly is the proper tool for exploring the universe as we perceive it at present."

The mathematics of peaks, passes and pits.

For networks with arcs, nodes and regions we have the well-known *Euler rule* $R + N = A + 2$ where R is the number of regions, N the number of nodes and A the number of arcs. For example, for the network below, $R = 4$ (including the region outside the network), $N = 5$ and $A = 7$.



And so $R + N = A + 2 (= 9)$. There is an analogy of this rule for polyhedra. If for any polyhedron V = number of vertices, F = number of faces and E = number of edges then $V + F = E + 2$.



For example for the cube shown above $V = 8, F = 6$ and $E = 12$. And clearly $V + F = E + 2$.

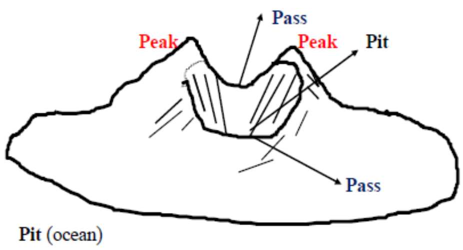
The *Euler rule* analogy for polyhedra is well known but what is not so well known is that this Euler rule has an equivalent in physical geography, providing we naturally accept that geographical terrain to have only three features:

Peaks: hills, mountains: these are points which are higher than any in the neighbourhood, all terrain slopes downwards from a peak.

Pits: hollows, valleys, volcanoes, lakes, sea: points which are lower than any in the neighbourhood, all terrain slopes upwards from a pit.

And **Passes**: a pass is the lowest point from which you can cross from one pit to the next.

Now if If P = number of peaks, Q = number of pits and R = number of passes then $P + Q = R + 2$. To illustrate this a simple island with representative pits, peaks and passes is shown below:



$P = 2$, $Q = 2$ and $R = 2$

Evidently $P + Q = R + 2$.

For an interesting analogue of the *Euler rule* in weather see: http://bit.ly/1fNac9V.

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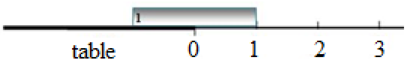
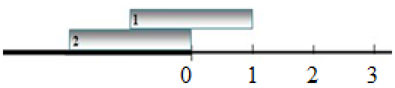
Staircase to heaven.

For this article we assume that the reader is familiar with A2 mechanics, in particular with the algorithm for calculating centres of mass.

Consider a book of length 2 units, called book 1, placed protruding as far from the edge (at 0) of a table so that is does not topple. The maximal position clearly is:



Now consider two identical books labelled 1 and 2. Place book 1 so that its centre of mass is just above the right edge of book 2:



Clearly book 1 will not topple over the right edge of book 2. But how far can this 2 stack configuration be moved to right before the 2 stack topples over the edge 0 of the table?

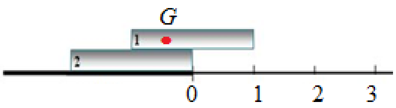
To work this maximal equilibrium position out we need to find the centre of mass G (measured from the 0 position) of the composite shape.

Using the following notation that book 1 = b1, book 2= b2, $\bar{x}$ the centre of mass from 0, M the moment about 0 and X the centre of mass of the composite object book 1 and book 2, the usual calculation table is:

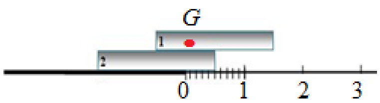
Object	b1	b2	b1+b2
Weight	1	1	2
$\bar{x}$	-1	0	X
M	-1	0	$2X$

Now the sum of the moments of the two books separately = the moment of the composite two books. That is, $-1 = 2X$. So $X = -\frac{1}{2}$.

So the centre of mass G of the composite two books is $\frac{1}{2}$ units to the left of 0 as shown below:

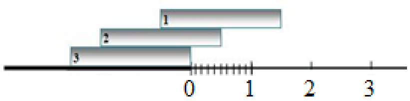


Therefore the 2 stack will rest in equilibrium provided the centre of mass G of the composite does not protrude past the edge. For the maximal position we can move the stack $\frac{1}{2}$ unit to the right with the centre of mass G directly above 0:



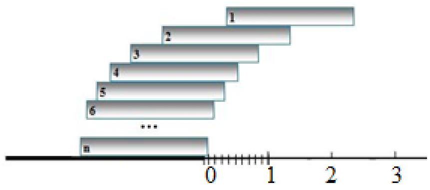
The overhang past the table edge is $d_2 = 1 + \frac{1}{2}$.

Now consider a 3 stack of books 1, 2 and 3 stacked above each other in such a way that book 1 does not topple over the edge of book 2, books 1 and 2 together do not topple over the edge of book 3. After calculating the centre of mass G_1 of the 3 stack, we find (figure below)that in the maximal equilibrium position over 0 the overhang is $d_2 = 1 + \frac{1}{2} + \frac{1}{3}$:



If we continue inductively in this way we will find that in the maximal equilibrium position the overhang d_n for n books is given by
$d_n = 1 + \frac{1}{2} + \frac{1}{3} + ... + \frac{1}{n}$.

A picture of n identical books stacked on top of each other so that they are in the maximal equilibrium position over the edge 0 is:



If we had an infinite time and staggered an infinite amount of books in the maximal equilibrium position then the overhang is

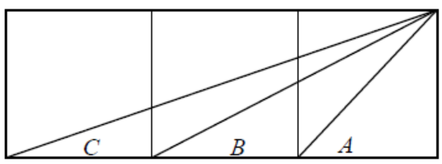
$d_\infty = 1 + \frac{1}{2} + \frac{1}{3} + ... + \frac{1}{n} + ...$

d_∞ is called the *harmonic* series which diverges, albeit slowly, to infinity. So, theoretically in the situation abovet we have built a balanced stack of books with infinite overhang and infinite height. A staircase to heaven!

However we need to be aware of the very, very slow rate that d_n diverges to infinity. For "even if a machine would have been adding terms at a rate of 10^{-9} seconds and would have started 15 billion years ago the value of the sum would still be about $\ln 10^{26}$ which is less than 60!" (see http://bit.ly/1tQxnTR).

Problem corner.

The diagram below shows three identical abutting squares and three angles A, B and C:



Without using trigonometry show that $A = B + C$.

The best solution(s) to this problem will be acknowledged in newsletter no. 6.

Solution to the problem in newsletter no. 4. A most elegant solution was provided by Steven Jin of Leicester Grammar school. See here: http://bit.ly/2tBAp85