University of Leicester

Nuffield Foundation

A guide to the use of practical tasks and manipulatives in the teaching of fractions and decimals with children aged 3 to 11

Examples for teachers

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CHILDREN HOLDING A HALF-PRICE SALE

Examples for Teachers

is one of three documents from our project developing guidance on teaching about fractions and decimals, which was funded by the Nuffield Foundation. The other documents are the Main Report and the Review of Research.

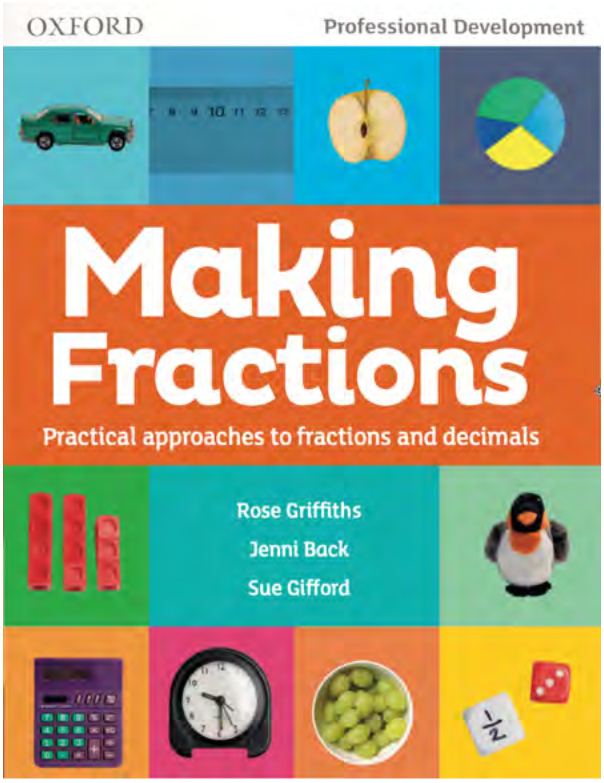
All three are available free on the Nuffield Foundation website:
https://www.nuffieldfoundation.org/project/teaching-fractions-and-decimals-to-children-aged-3-to-11

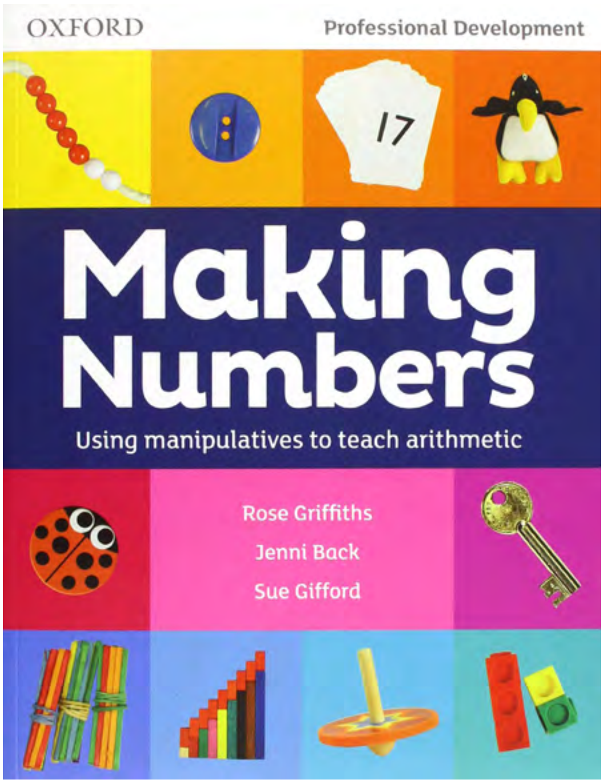
This document gives you a brief introduction to some key findings from our study and provides examples of the kinds of activities we recommend for learning about fractions and decimals.

We want children to see that fractions and decimals extend the number system in a helpful way.

Our complete guidance for teachers, published in 2023 by Oxford University Press, is ***Making Fractions:*** *practical approaches to fractions and decimals*.

Supporting this book, there are two lively animations to use with children, available free on the OxfordOwl website: *Jam tarts and halves and Decimal Dragon*.





Making Numbers

Our earlier guidance on effective and enjoyable ways of teaching arithmetic with whole numbers using practical materials is published by Oxford University Press (2016).

OUP's free website, OxfordOwl, hosts four animations for use with children, and four short professional development films to accompany this book.

The research and development leading to *Making Numbers* was also funded by the Nuffield Foundation. Our Main Report, Literature Review and Examples for Teachers are freely available to download on the Nuffield website: https://www.nuffieldfoundation.org/project/using-manipulatives-in-the-foundations-of-arithmetic-2

Introduction

Children who are confident with counting and making sense of whole numbers will discover that there are some problems that need a new kind of number, a fraction, to solve them.

Children's experiences of social sharing form the basis of an understanding of fractions as dividing into equal parts, building on ideas of fairness. Measuring for a purpose leads them into finding more accurate measurements, where they can see how fractions or decimals can help.

Common fractions:

$\frac{1}{2}$ $\frac{3}{4}$

Decimal fractions:

0.5 0.75

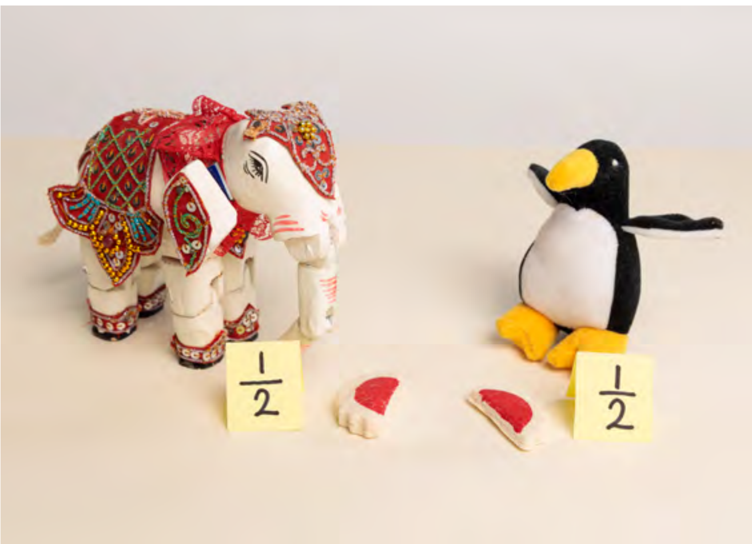
We usually just call these *fractions and decimals*

Our green car travelled 2.13 metres

In our earlier Nuffield Foundation funded project, our focus was on whole numbers and how to use manipulatives: items that you can handle and move which can help children solve problems. We suggested effective and creative approaches that would build children's understanding and skills.

Our approach to fractions and decimals is similar. Real-life and imaginary contexts help children to try out new ideas and to understand the maths they are using. They can use a range of strategies, including practical activity, talking, drawing and writing, and gradually move towards a confident use of more abstract ways of working.

In *Making Fractions*, we have organised two chapters to provide a flow of work for learning about common fractions, followed by two chapters for decimal fractions. In this report we will often look at fractions and decimals together, to emphasise the links between them.



Contents

Fractions and number sense

Number sense is sometimes called a ‘feel’ for number. There are many ways of defining it, all stressing that we want children to be able to think about ‘how numbers work’ in several different ways. When using whole numbers we looked at three key aspects of number sense, and these are useful when thinking about fractions, too.

COUNTING

knowing how fractions are named and how to say, read and write fractions.

understanding how to count, forwards and backwards, in ones and also in halves, quarters, tenths, hundredths and other fractions as needed

For example:

knowing that ‘a quarter’ can be written as $\frac{1}{4}$ and that 1.25 is said as ‘one point two five’.

counting in halves: $3\frac{1}{2}$, 3, $2\frac{1}{2}$, 2, $1\frac{1}{2}$, 1, $\frac{1}{2}$, 0

counting in tenths: 0.6, 0.7, 0.8, 0.9, 1, 1.1, 1.2, ...

COMPARING

knowing that if you divide something into more equal parts, each part will be smaller.

knowing that the size of a fractional part depends on the size of the ‘whole one’.

putting fractions in order of size.

estimating and rounding fractions (eg to the nearest whole number)

For example:

seeing that half of a big cake is not the same amount as half of a small cake.

knowing that $5\frac{3}{4}$ is closer to 6 than to 5.

knowing that 5.3 is bigger than 5.13

COMPOSITION

understanding how each number can be made in many different ways, including by using addition, subtraction, multiplication and division.

knowing that our decimal number system uses tenths and hundredths as well as hundreds, tens and ones.

making and finding equivalent fractions.

For example:

knowing that $2\frac{1}{2} = 1 + 1 + \frac{1}{2} = 5 \times \frac{1}{2} = 2 + \frac{1}{4} + \frac{1}{4}$

knowing that $\frac{1}{2}$ written as a decimal is 0.5 and $\frac{1}{4}$ is 0.25

knowing that $\frac{1}{2} = \frac{2}{4} = \frac{4}{8} = \frac{8}{16}$

Making connections

For some of us, learning about fractions and decimals was about learning rules, rather than understanding what we were doing. Turning this around so that understanding comes first may take a little longer, but rules alone are often forgotten or muddled up.

Engaging children's interest in solving a problem for themselves, with help from each other as well as from their teacher, builds confidence and develops their thinking. We want children to make mathematical connections – for example, to see a pattern, or to recognise that a new problem has things in common with one they have tried before.

Children working together

Working with a partner provides more opportunities to talk and to share ideas. Explaining your reasoning to someone else, including by showing them practically or talking through your writing or drawing, is an important skill that needs the opportunity to develop.

Concrete, pictorial, abstract

In maths we often have to make connections between real objects, drawings or mental images, and abstract symbols.

Ben's class worked on a problem where they had to share 27 grapes between four children.

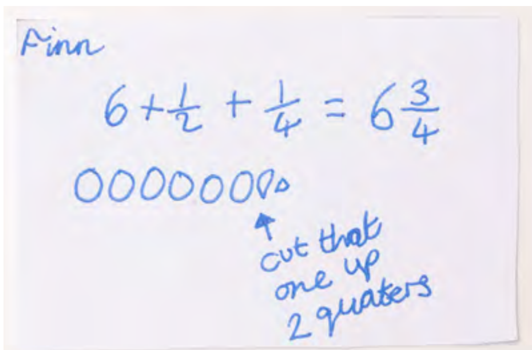
They used real grapes – and later on, they could use counters or cubes to stand for the grapes. *(Concrete items)*

They drew grapes, just as little oval shapes – or just imagined them. *(Pictorial)*

As their work continued, they began to use written numbers, too. *(Abstract)*

Many researchers have examined the links between practical activity and symbolic representation. Bruner[1] refers to enactive, iconic and symbolic modes of representation. Mason[2] interprets these as three different worlds of experience: moving between manipulable objects; mental imagery or drawing; and abstract symbols. It is not a 'one way street' from concrete to abstract – there are many times, for example, where it is helpful to move from an abstract to a drawn representation.

Finn's work on paper shows him using numbers (ie symbols) then pictures alongside each other, as he showed how he had added $\frac{1}{2}$ and $\frac{1}{4}$.



Footnotes:

(1) Bruner, J. (1966) *Toward a theory of instruction*. Cambridge, Massachusetts: Harvard University Press

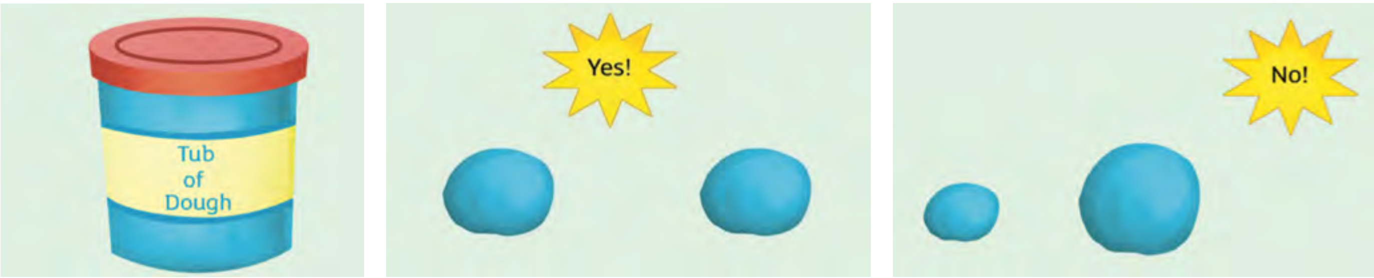
(2) Mason, J., Burton, L. and Stacey, K. (2010) *Thinking mathematically*. 2nd edition. Harlow: Pearson

Halves and mixed numbers

A half is probably the fraction that people use the most in everyday life. Many young children are familiar with halves, in several contexts. They might eat half a banana, know that they are $5\frac{1}{2}$ years old, be told 'we are half-way there', or that it is half-past seven. Working with halves can give them a confident understanding of many of the most important features of a fraction.

People do not always use words in daily life as accurately as in mathematics – even adults may say that they would like 'the bigger half'. One thing we need to establish with children is that dividing something in half gives us two parts of equal size. It helps to explore both what is and what is not a half.

Have we got half each?



We want children to realise that 'a half' can be many different things, depending on what it is half of. In particular, the size of a half depends on the size of the whole one: for example, half of a big lump of dough will be bigger than half of a small lump.

As part of our introduction to halves, children helped us make salt-dough jam tarts, to think about two problems.

1. What if you had two friends sharing jam tarts, and different numbers of jam tarts?
2. What if you had different numbers of friends sharing, but just six jam tarts?

The first problem shows that you can find half of any number of jam tarts. An even number of jam tarts is easy to share between two and will give a whole-number answer. An odd number of tarts will require cutting one jam tart in half.

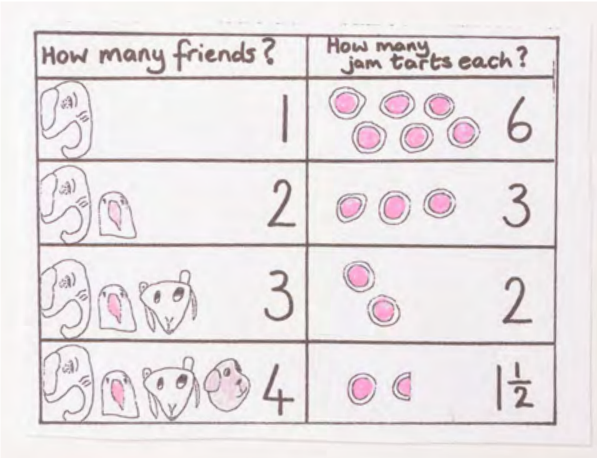

5 jam tarts, so $2\frac{1}{2}$ each

This activity gave children the chance to practice counting $\frac{1}{2}$, 1, $1\frac{1}{2}$, 2, $2\frac{1}{2}$, 3, and so on, and to see how to say, read and write those numbers – trying to write the '$\frac{1}{2}$' as close to the height of the whole number as they could. We use smaller numerals to write the fractional part, as it is not as big as a whole one.

Children could also look at how many halves you need to make each number:
for example, $1\frac{1}{2}$ = 3 halves = $\frac{3}{2}$ and 4 halves = $\frac{4}{2}$ = 2.

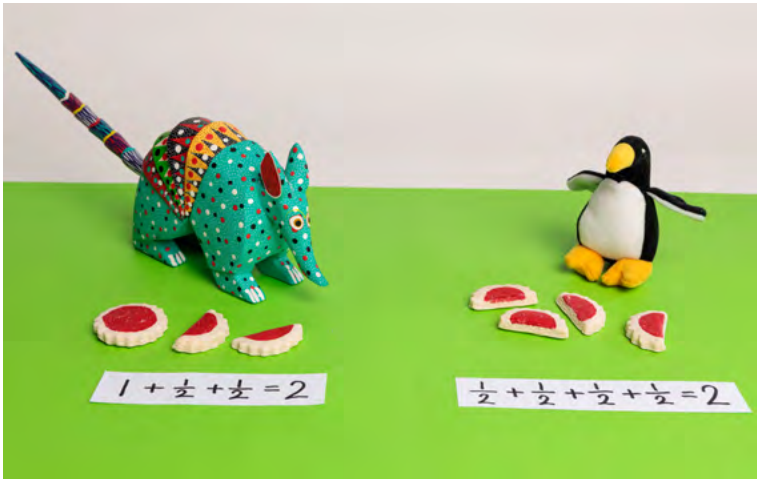
Ask children if they can see why $\frac{5}{2}$ is called a 'top-heavy' fraction. It can also be called an 'improper fraction'.

The second problem gave children in Jane's class a chance to see what happens if you have a growing number of animals sharing the same number of objects. Jane helped them compile a table of results, working systematically. As six jam tarts were shared between Elephant, Penguin, Sheep and then Bear, children could see that the shares became smaller.



How many friends?	How many jam tarts each?
1	6
2	3
3	2
4	$1\frac{1}{2}$

Each column in this table has a pictorial representation with the number written alongside it, helping children become more confident with the links between pictorial and abstract representations.



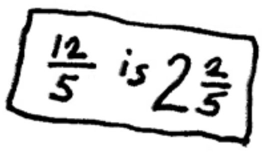
Exploring mixed numbers with halves helps children see that fractions exist throughout the number system, and not just between zero and one. Children can also see that a number can be composed in many different ways.

This early experience of mixed numbers and top-heavy fractions can be extended later on. For example, Karen asked the children in her class to choose: they could make each 'whole one' from either four or five bricks.

If my whole one is four bricks, each brick is worth a quarter.

If my whole one is five bricks, each brick is worth a fifth.

Karen asked, 'If five bricks was a whole one, what would 12 bricks make?'



They tried out some 'next-door' numbers of bricks, for example, 23, 24, 25, 26, then 27 bricks. For each number of bricks, they wrote the top-heavy fraction they had made, then turned that into a mixed number.

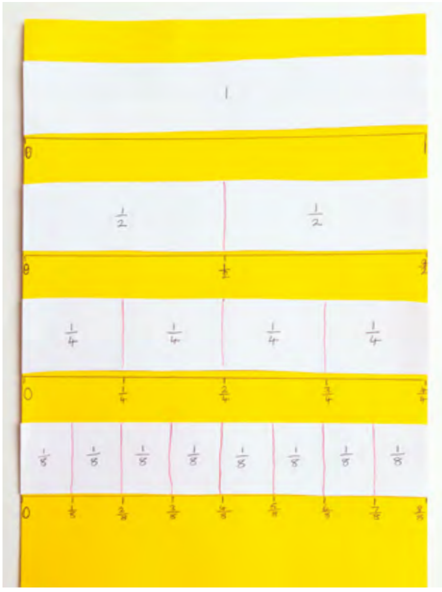
Working with sticks of five cubes, children began to get a sense of what was happening, and to generalise. For example, children said, 'You can't get more than four fifths left when it's in fifths' and 'There are no fifths left if the number of cubes is in the five times table.'

Working with fifths and tenths provides a strong link with decimals later on.

Families of fractions

Halves, quarters and eighths are part of a 'family' of fractions, which you can swap to make each other. Children will see this, if you start by cutting a whole one in half, then cut the halves in half again, to make quarters: $\frac{1}{2} = \frac{2}{4}$. If you cut the quarters in half, you will have twice as many equal pieces. Because there are eight pieces, they are called eighths: $\frac{1}{2} = \frac{2}{4} = \frac{4}{8}$

Spending time making, talking about, drawing and visualising the family of halves and quarters will build children's confidence and fluency. Then they will be ready to deal with other families of fractions.



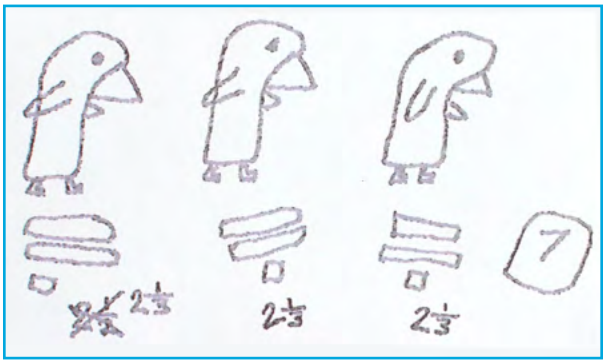
This fraction wall makes a direct link between fractions as part of the area of a rectangle, and fractions on a number line. Children can look for equivalences: for example, that $\frac{3}{4}$ is the same amount as $\frac{6}{8}$.

Importantly, they can see that two halves, four quarters and eight eighths all make one.

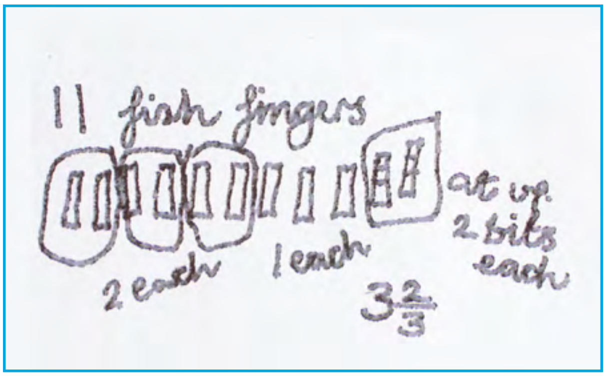
They can explore different ways of combining these fractions: for example, you can make $\frac{5}{8}$ with $\frac{1}{2} + \frac{1}{8}$, or with $\frac{1}{4} + \frac{1}{8} + \frac{1}{4}$. They can also visualise fractions bigger than one: 'How much would $\frac{5}{8}$ add $\frac{5}{8}$ make?'

Sharing between three leads naturally to working with thirds. Thirds can be split into sixths or ninths, making another family of fractions.

Vicki's class imagined there were three penguins sharing fish fingers. The children used orange kitchen sponge cloth to make 'fish fingers'. Vicki asked them to choose a number of fish fingers from four to 20, and to figure out how many each penguin would get.



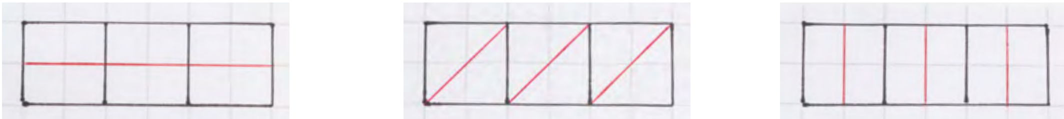
William and Fergus shared seven fish fingers between three penguins. They swapped the last fish finger for three thirds, then drew what they had done. Initially Fergus wrote $2\frac{1}{2}$, but changed that as soon as William asked 'Two and a half?'



Riley and Kit drew 11 fish fingers, then shared them out on their drawing: two for each penguin, then another one each, then they drew lines on the last two to show them cut into thirds.

Each group of children chose several numbers to explore, often using different methods each time. The mixture of using practical items, drawing, discussion and writing was very useful.

Cath's class extended their work by thinking about how to share one fish finger between six baby penguins. They explored different ways of cutting it into sixths.



Work with coloured rods helped children see how they could combine halves, thirds and sixths to make a whole one. They used the rod that is equivalent to six cubes as their whole one.

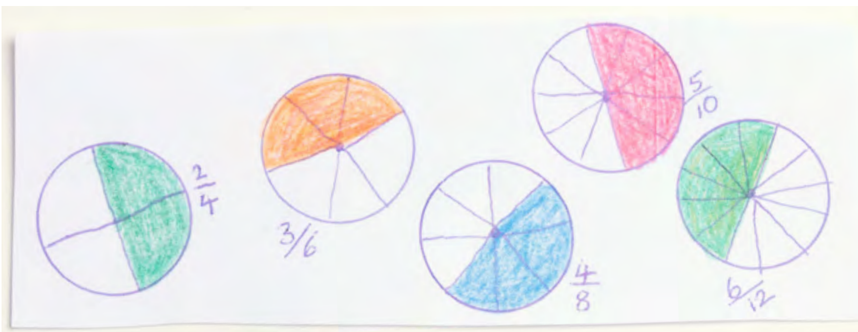
Half and a sixth and a third makes one!

Fifths and tenths (and hundredths) are part of another fraction family.

Rachael's class worked in pairs using small sticks on a drawn circle, to see if they could split the circle fairly into all the fractions from thirds to thirteenths, and sketched each division as they went.

Tilly and Jack made rough sketches as they went from $\frac{1}{3}$s to $\frac{1}{13}$s.

They looked for fractions that are the same as a half. They could see that you can make a half using three sixths or five tenths, but not from thirds or fifths, and they thought about why this happened.

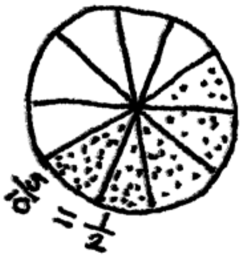


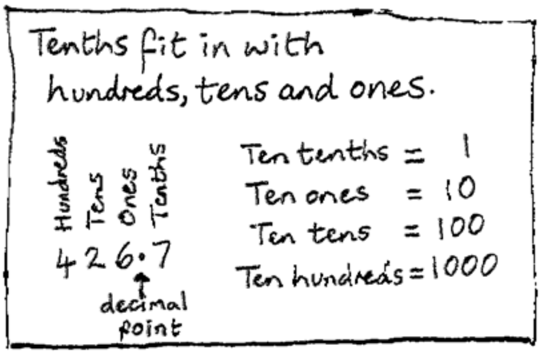
You can't make a half out of fifths. Because there's five of them, so you'd get three fifths on one side and two fifths on the other. No good.

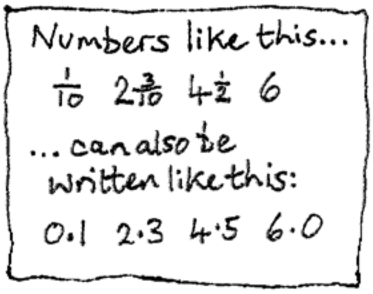
Mixed numbers and decimals

A half is a good place to start thinking about decimals. Dividing numbers by two in your head and then on a calculator lets children see how $\frac{1}{2}$ is written as a decimal. It also prompts two questions: 'Why is it 0.5?' and 'How does this fit in with the way we write whole numbers?'

Shabaana asked her class to think back to sharing different numbers of jam tarts between two. For example, if you had 7 jam tarts to share between two, they would get $3\frac{1}{2}$ each. On the calculator, this is 7 ÷ 2 = 3.5 Once the children had tried several examples, starting with any number of jam tarts that they wanted, they discussed why the calculator wrote a half as 0.5 and Shabaana could begin to help them make these important links:







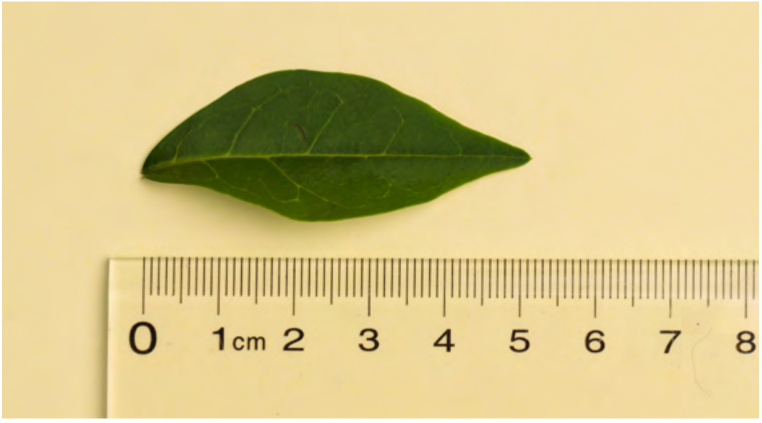
Practical measurement is a valuable context for decimals. Opinder's class started by seeing how far they could throw a bean bag, measuring to the nearest half-metre, and seeing that, for example, 2.5 metres = $2\frac{1}{2}$ metres. Later, they worked in threes to see how far they could jump.

When one person jumped, the other two would watch from each side and estimate the jump to the nearest tenth of a metre. Some groups used a measuring tape at a right angle to their start line; others chalked a measuring line. The children could see that measuring using a fraction meant that your measurement could be more accurate than just with whole numbers.

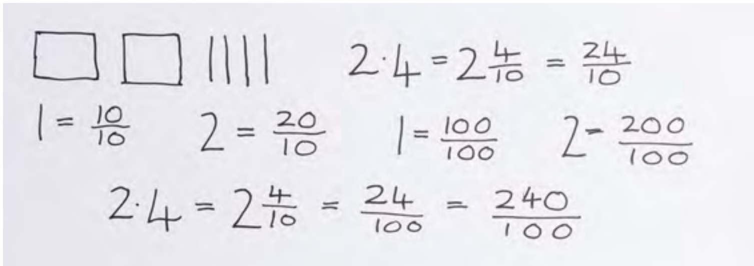
Our animation, *Decimal Dragon*, shows Dragon and Sheep measuring height to the nearest tenth of a centimetre, and weighing packets and tins to the nearest tenth of a kilogram. Children can think about the ways we can measure more accurately: either by using smaller units (for example, using grams as well as kilos, such as 1kg 400g) or by using a decimal (1.4 kg).

The calculator is an important tool for learning about decimals. To use a calculator, we need numbers written like 3.5, not $3\frac{1}{2}$, and a calculator helps us explore number problems and patterns. For example, 'See what happens when you multiply by 100. Try it with lots of different numbers'. Or work the other way around: 'How many ways can you find to make the calculator show 0.5?'

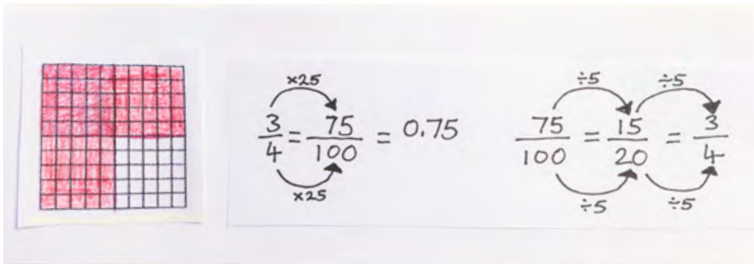
Etienne's class used calculators when they were learning about finding averages. Each group of children collected nine leaves, in a range of sizes, from the privet hedge around the playground. They sorted them in order of size and measured them to the nearest tenth of a centimetre. Finding the mean length using their own real data was motivating, and the calculator kept their attention on the process of averaging.



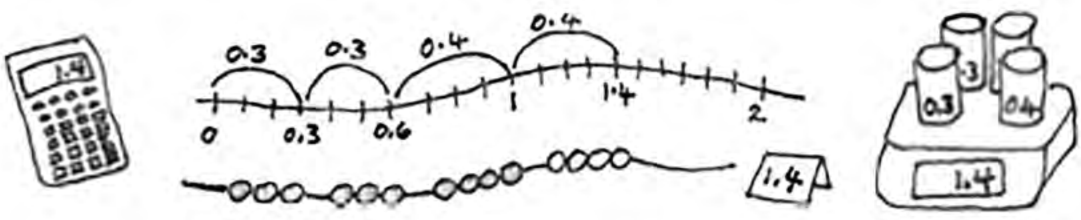
Repurposing hundreds, tens and ones equipment as ones, tenths and hundredths can be very powerful. When the square is one whole unit, then each 'stick' is 0.1 (a tenth). Each little cube is then a hundredth.



Both mental arithmetic and pencil-and-paper calculations benefit from children having a strong sense of how big an answer should be. Finding more than one way to reach an answer helps this process. For example, turning $\frac{3}{4}$ into a decimal, or 0.75 into $\frac{3}{4}$, can be shown by colouring a 10 by 10 hundred square. Once a child is sure that $\frac{3}{4}$ = 0.75, the standard method is easier to follow.



Children need to build a repertoire of different ways of approaching a calculation, including practical, drawn, mental and calculator methods. We know that each method and representation can support the understanding of another, so we should often ask, 'Is there another way you could do this?' Our aim is that children should be able to use fractions and decimals flexibly and confidently, building understanding through discussion and experimentation.



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